Macroeconomics - Spring Semester - QEM 1
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Perfect Competition and Fixed Prices

1 A Simple Model with Perfect Competition

Static model with three goods, a final good, labor and money, three types of agents, producers, consumers and a government, and perfect competition, i.e. all agents are price-takers and prices are flexible.

1.1 Production Sector

Assuming a representative firm, the production is given by $Y = F(N)$, with Y output and N employment.

Assumption 1 *$F(N)$ is a continuous function on $\mathbb{R}_+$, with $F(0) = 0$, two-times differentiable on $\mathbb{R}_{++}$, with $F'(N) > 0$, $F''(N) \leq 0$, $\lim_{N\to 0} F'(N) = +\infty$ and $\lim_{N\to +\infty} F'(N) = 0$.*

Profits are defined by $\Pi = PY - WN$, with P the price of the final good and W the nominal wage.

Maximizing profits, one obtains the labor demand: $W/P = F'^{d} = F'^{-1}(W/P)$, decreasing in the real wage W/P ($dN/d(W/P) = 1/F''(N)$).

We deduce the supply of final good: $Y = F[F'^{-1}(W/P)]$.

Example: Using $F(N) = N^a$, with $a \in (0, 1]$, $W/P = aN^{a-1} \Leftrightarrow N = (aP/W)^{1/(1-a)}$ and $Y = (aP/W)^{a/(1-a)}$.

1.2 Consumers

The preferences of a representative household are summarized by the utility function:

$$U(C, M/P, H - N) = \left(\frac{C}{b/(b+d)}\right)^b \left(\frac{M/P}{d/(b+d)}\right)^d - \theta\frac{N^e}{e} \tag{1}$$

with C the consumption, M/P the money demand (in real terms), H the time endowment, $\theta > 0$, $e \geq 1$, $b, d \in [0, 1]$, and $b + d \leq 1$.

The representative household maximizes his utility function facing the budget constraint:

$$PC + M = M_0 + \Pi - T + WN \tag{2}$$

with $M_0 > 0$ the stock of money and $T \geq 0$ lump-sum taxes. Taking as given the real income $I = M_0/P + (\Pi - T)/P + (W/P)N$, we obtain:

$$C = \frac{b}{b+d} I \text{ and } \frac{M}{P} = \frac{d}{b+d} I$$

We further note that when $b + d = 1$, $C = bI$ and $M/P = (1 - b)I$.

Then, labor supply is defined by:

$$\max_N \; I^{b+d} - \theta N^e / e \tag{3}$$

One obtains:

$$\frac{W}{P} = \frac{\theta}{b+d} N^{e-1} I^{1-b-d} \tag{4}$$

When $b+d = 1$, $W/P = \theta N^{e-1} \Leftrightarrow N = \left(\frac{1}{\theta}\frac{W}{P}\right)^{1/(e-1)}$, i.e. $1/(e-1)$ represents the elasticity of labor supply with respect to the real wage.

1.3 Government

Public expenditures (in real terms) are given by $G \geq 0$ and the budget is balanced $PG = T$.

1.4 Equilibrium

Using $\Pi/P = Y - (W/P)N$ and the balanced-budget rule $G = T/P$, the real income can be rewritten:

$$I = \frac{M_0}{P} + Y - G \tag{5}$$

Therefore, the equilibrium on labor market is given by:

$$\frac{W}{P} = aN^{a-1} = \frac{\theta}{b+d} N^{e-1} \left(\frac{M_0}{P} + Y - G\right)^{1-b-d} \tag{6}$$

with $Y = N^a$, and the equilibrium on the product market by:

$$\begin{aligned} C = Y - G &= \frac{b}{b+d}\left(\frac{M_0}{P} + Y - G\right) \\ \Leftrightarrow P &= \frac{b}{d}\frac{M_0}{Y - G} \end{aligned} \tag{7}$$

Equilibrium on money market is ensured by Walras law.

1.5 Case $b + d = 1$

$$\begin{aligned} N^{pc} &= \left(\frac{a}{\theta}\right)^{1/(e-a)} , \quad \left(\frac{W}{P}\right)^{pc} = a^{\frac{e-1}{e-a}}\theta^{\frac{1-a}{e-a}} \\ Y^{pc} &= \left(\frac{a}{\theta}\right)^{a/(e-a)} , \quad P^{pc} = \frac{b}{1-b}\frac{M_0}{Y^{pc} - G} \end{aligned} \tag{8}$$

Money and public spending are neutral. $\Delta M_0 > 0$ and $\Delta G > 0$ only imply $\Delta P^{pc} > 0$, without effect on Y^{pc}, N^{pc}, and on the relative prices.

1.6 Case $b + d < 1$

Substituting (7) into (6), we get:

$$\frac{\theta}{a(b+d)} N^{e-a} \left[\frac{b+d}{b}(N^a - G)\right]^{1-b-d} = 1 \tag{9}$$

Differentiating, we obtain:

$$\frac{dN}{dG} = \frac{(1-b-d)N}{N^a[e - a(b+d)] - G(e-a)} > 0 \tag{10}$$

Using $Y = N^a$,

$$\frac{dY}{dG} = \frac{(1-b-d)aY}{N^a[e - a(b+d)] - G(e-a)} \in (0,1) \tag{11}$$

The economic mechanism is based on an income effect that affects the labor supply when $b + d < 1$. $G \uparrow \Rightarrow T \uparrow \Rightarrow$ income $\downarrow$ $\Rightarrow$ labor supply increases for all level of W/P. Therefore, at equilibrium, W/P decreases.

Since $I = \frac{b+d}{b}(Y - G)$, we get $dI/dG < 0$, i.e. an increase of G reduces C and M/P.

Using (7) and (9), one may also conclude that monetary policy is neutral.

One attempt to have keynesian results in macroeconomic models with micro-foundations is based on price rigidities: the fixed-price approach.

2 The Model with Fixed Price and Wage

Preliminaries: Consider a single good market characterized by a demand $Y^d = D(P)$ and a supply $Y^s = S(P)$, with $D'(P) < 0$ and $S'(P) > 0$. A competitive equilibrium is defined by (Y^*, P^*) satisfying $Y^* = D(P^*) = S(P^*)$. Under a fixed price P generically different from P^*, we have $Y^s \neq Y^d$. In such a case, the quantity is rationed and is given by $Y = \min\{Y^s, Y^s\}$, i.e. $Y = Y^s$ if $P < P^*$ and $Y = Y^d$ if $P > P^*$.

Considering now the model of the previous section with $b + d = 1$, we have:

$$\begin{aligned} N^d &= (aP/W)^{1/(1-a)} \ , \ N^s = \left(\frac{1}{\theta}\frac{W}{P}\right)^{1/(e-1)} \\ Y^d &= \frac{b}{1-b}\frac{M_0}{P} + G \ , \ Y^s = \left(a\frac{P}{W}\right)^{a/(1-a)} \end{aligned} \tag{12}$$

where the perfectly competitive equilibrium $(Y^{pc}, N^{pc}, P^{pc}, W^{pc})$ is given by (8).

Assuming that W and P are fixed and (P, W) is generically different from (P^{pc}, W^{pc}), quantities are rationed. Then, at equilibrium, the level of employment and product are determined by:

$$N = \min\{N^d, N^s\} \ , \ Y = \min\{Y^d, Y^s\} \tag{13}$$

Therefore, we are able to define the following typology of equilibria:

- $Y^d < Y^s$ and $N^d < N^s$: keynesian unemployment;
- $Y^d > Y^s$ and $N^d < N^s$: classical unemployment;
- $Y^d > Y^s$ and $N^d > N^s$: repressed inflation;
- $Y^d < Y^s$ and $N^d > N^s$: not relevant.

2.1 Keynesian Unemployment ($Y^d < Y^s$ and $N^d < N^s$)

We have $Y = \frac{b}{1-b}\frac{M_0}{P} + G$ and $N = F^{-1}(Y)$.

One need $Y^d < Y^s$, i.e.

$$\frac{b}{1-b}\frac{M_0}{P} + G < \left(a\frac{P}{W}\right)^{a/(1-a)} \Leftrightarrow \frac{W}{P} < a\left(\frac{b}{1-b}\frac{M_0}{P} + G\right)^{-(1-a)/a} \tag{14}$$

Moreover, such equilibrium is such that $N^d < N^{pc}$ ($< N^s$), i.e.

$$F(N^d) < F(N^{pc}) \Leftrightarrow \frac{b}{1-b}\frac{M_0}{P} + G < \frac{b}{1-b}\frac{M_0}{P^{pc}} + G \Leftrightarrow P > P^{pc} \tag{15}$$

- A decrease in W has no effect.
- A decrease in P stimulates output and employment.
- Keynesian multiplier, $dY/dG = 1$.

2.2 Classical Unemployment ($Y^d > Y^s$ and $N^d < N^s$)

The producer does not perceive any quantity rationing, i.e. $F'(N) = W/P$. This requires $Y^d > Y^s$, i.e.

$$\frac{W}{P} > a\left(\frac{b}{1-b}\frac{M_0}{P} + G\right)^{-(1-a)/a} \tag{16}$$

and also $N^d < N^{pc}$ ($< N^s$) which is equivalent to:

$$F'^{d}) > F'^{pc}) \Leftrightarrow \frac{W}{P} > \left(\frac{W}{P}\right)^{pc} \tag{17}$$

- Increasing demand (G) has no effect on output and employment. Since $C = Y - G$, this only decreases private consumption.
- Only a decrease of W/P can increase the level of output Y^S and restore full employment.

2.3 Repressed Inflation ($Y^d > Y^s$ and $N^d > N^s$)

In this regime, we have $N^d > N^{pc}$ ($> N^s$), i.e.

$$F'^{d}) < F'^{pc}) \Leftrightarrow \frac{W}{P} < \left(\frac{W}{P}\right)^{pc} \tag{18}$$

and $Y^d > Y^{pc}$ ($> Y^s$), which is equivalent to:

$$\frac{b}{1-b}\frac{M_0}{P} + G > \frac{b}{1-b}\frac{M_0}{P^{pc}} + G \Leftrightarrow P < P^{pc} \tag{19}$$

2.4 Concluding Remarks

- One obtains a classification with clear-cut policy recommendations.
- However, such type of models have a crucial weakness: the absence of a satisfactory theory of price and wage formation.
- Therefore, another attempt to have economic models with market failures and (perhaps!) keynesian features: macroeconomic models with imperfect competition $\rightarrow$ explicit price and wage formation.