Welfare Cost of Fluctuations When Labor Market Search Interacts with Financial Frictions

Eleni Iliopulos* François Langot† Thepthida Sopraseuth‡§

Submitted : February 2017. Revised: September 2018

Abstract

We study the welfare costs of business cycles in a search and matching model with financial frictions à la Kiyotaki & Moore (1997). We investigate the mechanisms that allow the model to replicate the volatility on labor *and* financial markets and show that business cycle costs are sizable. We first demonstrate that the interactions between labor market and financial frictions magnify the impact of shocks via *(i)* a credit multiplier effect and *(ii)* an endogenous wage rigidity inherent to financial frictions. Secondly, in a non-linear framework, we show that the large welfare costs of fluctuations are also explained by the high average unemployment and the low job finding rates with respect to their deterministic steady-state values.

JEL Classification : E32, J64, G21

Keywords: Welfare, business cycle, financial friction, labor market search

*U. of Evry. eleni.iliopulos@univ-evry.fr

†U. of Le Mans (GAINS-TEPP & IRA), Paris School of Economics and IZA. flangot@univ-lemans.fr

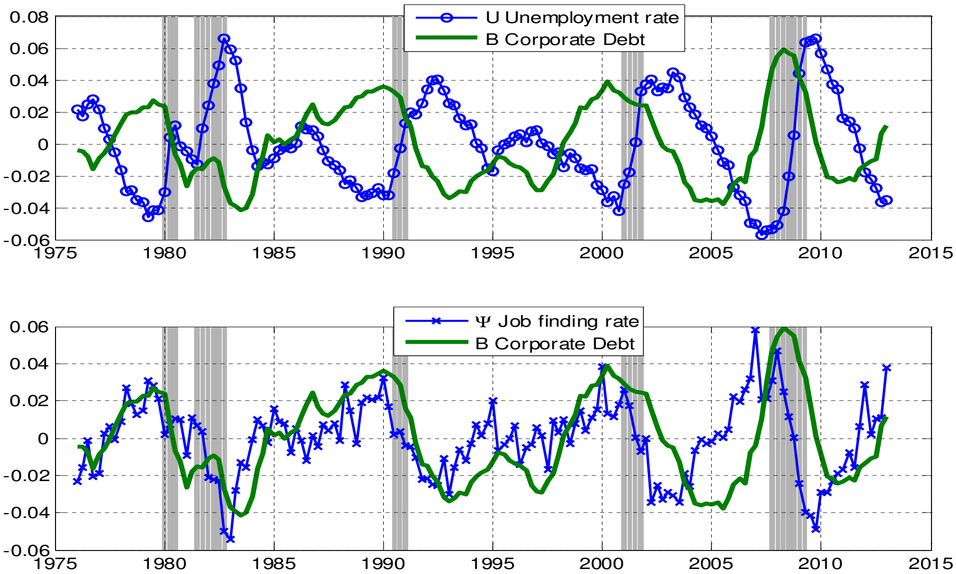
‡U. of Cergy Pontoise (THEMA). thepthida.sopraseuth@u-cergy.fr

§We thank Guido Ascari, Paul Beaudry, Florin Bilbiie, Matteo Cacciatore, Daniel Cohen, Martin Ellison, Andrea Ferrero, Pietro Garibaldi, Céline Poilly, Nicolas Petrosky-Nadeau, Vincenzo Quadrini, Xavier Ragot, Federico Ravenna, Gilles Saint-Paul, Etienne Wasmer, Pierre-Olivier Weill, Francesco Zanetti for helpful comments as well as seminar participants at Paris School of Economics macro-workshop, Cepremap, University of Lille I, University of Lyon - Gate, HEC Montreal, T2M Conference, Search and Matching Annual Conference, ScPo Conference in honor of C. Pissarides, TEPP conference, SED conference, Oxford macroeconomics seminar. E. Iliopulos and T. Sopraseuth thank Labex MME-DII (ANR-11-LBX-0023-01). T. Sopraseuth acknowledges the support of the *Institut Universitaire de France*

1 Introduction

This paper investigates the welfare costs of fluctuations in an original framework that features strong interactions between financial and labor market frictions. A glance at the data suggests the empirical relevance of this interaction. Figure 1 shows that episodes of job creation (when the job-finding rate Ψ rises) are also times when firms accumulate debt.[1]

Figure 1: Cyclicality of unemployment rate U, job finding rate Ψ and debt stock B.



HP Filtered logged quarterly data. Shaded area shows recessions (NBER dates). HP filtered data on U and Ψ were divided by 4 for the purpose of scale consistency. Smoothing parameter: 1600. Source : See Appendix A.

We argue that the strong interactions between financial and labor frictions can potentially lead to sizeable welfare costs of business cycles. This results does not only come from the rise of aggregate volatility, but also from the increase in the gaps of unemployment and job finding rates with respect to their deterministic steady-state values. Indeed, if unemployment increases during recessions outweigh falls in unemployment during economic booms, average unemployment (hence average consumption) lies above (lies below) its steady-state counterpart, such that welfare costs of fluctuations becomes sizeable. Obviously, a satisfactory assessment of welfare costs of the business cycle must rely on *(i)* a model that replicates the volatility of macroeconomic aggregates (quantities and prices) and *(ii)* a quantitative approach that accounts for model non-linearities so as to distinguish the averages from the steady-state values of macroeconomic aggregates. This paper analyzes these points using a Diamond (1982)–Mortensen (1982)–Pissarides (1985) (hereafter DMP) model in which en-

[1] Jermann & Quadrini (2012) also notice that debt repurchases (a reduction in outstanding debt) increase during or around recessions.

trepreneurs' access to credit is limited by a collateral constraint as in Kiyotaki & Moore (1997), because of enforcement limits. Agents are characterized by heterogeneous discount rates: impatient financially-constrained entrepreneurs borrow from patient (saver) households. Entrepreneurs borrow so as to finance intra-period hiring costs. This gives rise to an interaction between labor market and financial frictions. Moreover, because of Kiyotaki & Moore (1997)'s setting, the wage-bargaining process is characterized by original features that entail labor-market fluctuations to match the business cycle.

Beyond the usual "financial accelerator" effect inherent to DSGE models with financial frictions[2], we show that the *interactions* between financial and labor frictions generate endogenous real wage rigidity that magnifies in turn the impact of shocks. This allows us to solve the Shimer (2005) volatility puzzle. The intuition is the following. In booms, the cost of credit falls so that firms enjoy a lower cost of hiring and open more vacancies. This is the so-called "credit multiplier" found in the literature.[3] [4] The opening of new vacancies also increases the average duration to fill a vacancy (due to congestion effects). This raises the quantity of credit the firm needs to borrow, which is proportional to the duration to fill a vacancy. Notice that in our model the share of the match surplus used to finance credit costs is perceived to be lower by the (impatient) entrepreneur than by the (patient) worker. This is a key element to make the entrepreneur reluctant to accept wage increases in booms. We show that this counter-cyclical component in the wage setting rule is the channel through which our model can generate endogenous wage rigidity. Rigid wages are consistent with the data and entail in turn large fluctuations in labor-market aggregates that match the empirical volatility.[5]

From a normative point of view, we investigate the non-linear mechanisms by which economic losses in recessions are larger than gains from booms. We first derive analytical results based on a simplified version of the model. The DMP model is *per se* potentially able to predict unemployment increases in recessions that are larger than unemployment falls in booms. This explains why in our model average unemployment is higher than its deterministic counterpart throughout the cycle. Indeed, the Beveridge curve implies that unemployment

[2]See Gertler & Kiyotaki (2010) for a survey.

[3]See Petrosky-Nadeau (2013) among others.

[4]Notice that there is only a technological shock in our model. This restrictive view of the sources of fluctuations has a double advantage: first, it allows us to isolate the mechanism at work in the model; second, it facilitates the comparisons of our framework with the other contributions on the Shimer (2005) puzzle. We include financial shocks as a robustness check in section 5.4.

[5]This wage rule is thus more relevant with respect to the introduction of rigid wages. Wage volatility in US data is 60% that of output. As Chéron & Langot (2004) and Pissarides (2009) that echo the old Keynes-Tarshis-Dunlop controversy, we stress the need to understand fluctuations in both unemployment and wages.

is a convex function of the job finding rate. Thus, fluctuations in the job finding rate push the average rate of unemployment above its deterministic steady-state value. Notice that this feature is shared by all DMP models.[6] The large size of fluctuations in the job finding rate[7] is not sufficient to generate our results. Indeed, the gap between the average job finding rate and its deterministic steady-state value is also crucial for the evaluation of welfare costs of business cycle. We show that two opposite forces are at work.

First, one force dampens welfare costs because hiring decisions are a convex function of aggregate productivity. This leads the standard stochastic DMP model to generate a higher average level of labor-market tightness with respect to its deterministic counterpart, thereby dampening the welfare costs of fluctuations. As unemployment falls during booms, hiring takes more time so that firms create more vacancies to reach their targeted employment level. Labor market tightness hence increases more in expansion than it declines in recession. Moreover, given that the matching function exhibits decreasing marginal returns to vacancies, the job-creation condition is satisfied for greater variations in job creations during booms than during recessions.

A second opposite force, which is specific to our model, magnifies welfare costs by generating a concave relationship between hiring decision and productivity. The key element in the model is the new component specific to financial frictions, which is incorporated into the wage equation. In booms, large increases in labor market tightness also mean that it takes longer to hire workers. As firms use loans to pay for hiring costs, in booms, credit costs increase. This is taken into account in the bargained wage because *impatient* entrepreneurs perceive less future profits with respect to *patient* workers. Given that credit costs increase in booms, they significantly moderate wages hikes in expansion. As credit costs increase more in booms than it declines in recession, smaller variations in labor-market tightness are needed in booms than in recessions to satisfy the job creation condition. This mechanism can lead labor-market tightness to be a concave function of productivity and thus widens the gap between average and deterministic steady-state values of unemployment. We demonstrate that because of financial frictions *à la* Kiyotaki & Moore (1997) and their effect on the Nash bargaining, the model entails large welfare costs of business cycles.[8]

[6] See Yashiv (2007) for a survey.

[7] See the above discussion on the Shimer (2005)'s puzzle.

[8] One might be surprised that, in a model with financial frictions, credit costs expand in booms. We are referring here to the duration of credit, captured by the average time needed to fill a vacancy. The "credit multiplier" effect, that improves firms' access to credit in booms, is nonetheless at work in the model. We also show that financial frictions with rigid wages can even lead to welfare *gains* from fluctuations. Hence, wage adjustments in the Nash-bargaining rule play a crucial role in the understanding of sizeable welfare costs of cycles.

We build a calibrated DSGE model incorporating these features. We find that the introduction of financial frictions make the relative volatility of the job-finding rate twice as large with respect to what we obtain with a DSGE model with labor-market frictions only. By dampening wage increases, financial imperfections allow firms to preserve their incentive to open new vacancies in booms. This mechanism helps the model solve Shimer (2005)'s puzzle so that our unemployment and job finding rates are respectively 5.5 and 6 times more volatile than output and wage volatility is close to the data.[9] The model also correctly predicts the volatility on financial markets. As land supply does not vary over the business cycle, any change in land demand results in changes in the real-estate price. The model is then able to generate volatile collateral price (about 3 times more volatile than output). The predicted debt volatility is also consistent with the data (about 1.5 times more volatile than output). This is due to the interaction between financial and labor market frictions in the borrowing constraint. Indeed, absent labor-market frictions in the borrowing constraint, the volatility of the collateral price directly predicts the volatility of corporate debt. In this case, the model would predict a debt volatility that is too large with respect to data. When vacancies are incorporated into the borrowing constraint, the variance of the collateral price equals the sum of the variance of debt, that of vacancies and the covariance between debt and vacancies. In spite of a large volatility of the collateral price, because vacancies are very volatile and covariate positively with debt, our model is able to match the volatility of debt. The welfare cost of the business-cycle in a economy with financial and labor market frictions is 2.5% of workers' permanent consumption. It drastically falls without financial frictions and labor market frictions only (0.12%). These costs are far larger than the estimates by Lucas (1987, 2003), who reports a welfare cost of 0.05% in the case of logarithmic utility.[10]

The paper is organized as follows. Section 2 outlines our original contribution with respect to the literature. Section 3 describes the model. Section 4 uses a simplified version of the model to provide economic intuitions. Our quantitative analysis is developed in section 5. Section 6 concludes the paper.

[9]One could argue that wage rigidity is enough to solve the difficult exercise of obtaining a volatile labor-market response to productivity shocks. We stress that wage rigidity cannot be a relevant solution because wage volatility in US data is 60% that of output. Thus, wages do fluctuate throughout the business cycle.

[10]In order to show that this result is robust, we propose several extensions of our basic model in section 5.4

2 Related literature

Our contribution lies in bridging the gap between two strands of the literature, the first one studies the welfare costs of fluctuations and the second one investigates the macroeconomic impact of financial frictions on labor market dynamics. While each element has been studied independently, the contribution of our paper lies in combining both.

Lucas (1987, 2003) shows that welfare costs associated with business cycles are negligible. In his framework, costs associated with recessions are indeed compensated by the gains during booms. In this paper, we challenge Lucas's result by proposing a non-linear DSGE model that can generate significant asymmetries at business-cycle frequency. In our model, the costs of recessions cannot be compensated by the gains of expansions because average employment and average consumption are significantly lower than their deterministic steady-state levels. Welfare costs of fluctuations can then be significantly greater than those found by Lucas.[11] As explained by Hairault et al. (2010) and Jung & Kuester (2011), this is due to the fact that the non-linearities of the DMP model introduce a gap between the deterministic steady-state and mean unemployment levels. They show how this generates business-cycle costs ranging between 0.2% and 1.2% of permanent consumption in their calibrated models.[12]. However, these works neglect financial frictions. [13]

The link between the matching model and financial imperfections is intuitive.[14] Search activity is costly for firms and needs to be financed with borrowing. However, this investment cannot be used as a collateral. Our paper also complement the view developed by Hall (2017): unemployment increases in periods of high discount rates. In our framework, it is not the absolute value of the discount rate that matters but the gap between borrowers' (entrepreneurs) and lenders' (households) discount rates. Our general equilibrium approach

[11]In case of a significant gap between average and deterministic steady-state values, such costs are of a first-order magnitude – as the costs of tax distortion also evaluated by Lucas (1987).

[12]Our study differs from previous works on business-cycle costs, such as Beaudry & Pages (2001), Storesletten et al. (2001) or Krebs (2003), in that it regards interactions between the labor and financial frictions as the root of the welfare cost of business cycles.

[13]The asymmetries and non-linearities of labor markets have been also recently studied by Petrosky-Nadeau & Zhang (2017) and Ferraro (2018). Moreover, in Petrosky-Nadeau et al. (2018) the asymmetries charachterizing the DMP model together with cumulatively large negative shocks can explain rare events like disasters, as measured at the aggregate level by Barro (2006). Our paper complements these studies by analyzing the implications on welfare of the non-linearities in the DMP model.

[14]As suggested by Pissarides (2011), the "*equilibrium matching models are built on the assumption of perfect capital markets. ... But future work needs to explore other assumptions about capital markets, and integrate the financial sector with the labour market. This might suggest another amplification mechanism for shocks, independently from wage stickiness or fixed costs*" (Pissarides (2011)). Dromel et al. (2010) show that Pissarides (2011)' intuition is supported by significant empirical links between unemployment dynamics and financial frictions.

also explains the dynamics of this gap (the financial wedge) in a framework where fluctuations are driven by a technological shock.[15]

Recent research has studied how evolving conditions on credit markets affect the dynamics of labor markets. Many of them specifically focus on how to improve the ability of standard search models to match data and solve the Shimer puzzle. Petrosky-Nadeau (2013) introduces financial frictions *à la* Bernanke et al. (1999) in a DMP model. However, in order to match data, an ad hoc countercyclical monitoring cost is included in the financial contract. Garin (2015) analyzes the effect of a credit shock and argues that it is critical to understand labor market dynamics. However, he does not control for the implications on financial aggregates: in response to the shock, the debt volatility generated by his model is five times as greater as the one of output (see his Figure 3, p.122). This is counterfactual (see Table 2 in this paper). Liu et al. (2016) include housing-land holdings into the utility function so as to generate countercyclical movements in workers' outside options. However, their model yields limited interactions between labor and financial frictions as labor market variables do not enter the collateral constraint. This can lead them to overestimate the weight of the financial shock, necessary for them to fit the data.[16] In contrast to this work, Chugh (2013) includes wage payments into the working capital (together with capital rental payments). This model with financial frictions matches the volatility of the aggregates emphasized by Shimer (2005). However, as in all the previous papers, Chugh (2013) does not discuss the implications of his model with respect to real wage dynamics (the Keynes-Tarshis-Dunlop controversy). This is done in our work.

We contribute to this literature by using a streamlined model à la Kiyotaki & Moore (1997). We show that the interaction between financial and credit frictions does not systematically generate a strong propagation mechanism. We underline why in our framework financial frictions generate an amplification effect, which depends both on a credit-multiplier and a new amplification mechanism. Our original mechanism affects the wage-bargaining process and is specific to financial frictions *à la* Kiyotaki & Moore (1997). Our quantitative results suggest that these mechanisms are sufficient to solve the volatility puzzle of the DMP model.

[15]We depart from the DMP models with borrowing constraints à la Aiyagari (1994). Indeed, Krusell et al. (2010) shows that the averages of the aggregates generated by an economy where utility is linear are "remarkably similar" to those of their model with incomplete-markets and log utility (See table 5, p.1492, of the Krusell et al. (2010) paper). Given that the gap between average values and their deterministic steady-state counterparts are very close in an economy with linear utility and flexible wages (See Hairault et al. (2010)), we deduce that financial constraints *à la* Aiyagari (1994) do not generate welfare costs of the business cycle. In contrast, we show that the type of financial frictions we introduce, significantly increase the gap between the averages and the steady-state values of the aggregates, generating significant welfare costs of the business cycle.

[16]Moreover, they do not analyze the volatilities of quantities and prices as their estimation includes the real land price, but not corporate debt.

Therefore, it is not necessary to combine exogenous fluctuations in collateral requirements with financial constraints à la Kiyotaki & Moore (1997) to close the gap between the model and the data, as suggested by Liu et al. (2016) or Garin (2015).[17] We do not introduce a banking sector à la Gertler & Kiyotaki (2010)) in a DMP model, as done by Millard et al. (2017). In our paper, Kiyotaki & Moore (1997)'s setting introduces an heterogeneity in discount rates into the wage bargaining, which allows us to derive an explicit interaction between financial and labor frictions.

3 The model

The economy is populated by two types of agents: firms and workers. The representative firm produces the final consumption good of the economy by combining labor and infrastructure (i.e., land). Firms can finance their activity with loans funded by households. As debt contracts are not complete because of enforcement limits *à la* Kiyotaki & Moore (1997), firms are subject to a collateral constraint. Households include both unemployed individuals and workers employed by the firms. A canonical matching process *à la* Diamond-Mortensen-Pissarides allows firms to hire workers. Wages are set according to a standard Nash bargaining process. In order to stress the economic mechanisms at work, we present a streamlined model without capital accumulation: even without capital, households can save by lending to firms. [18] We lay stress on the extensive margin of labor, thereby discarding adjustments in hours, as in Blanchard & Gali (2010). Finally, we consider only technological shocks to allow for a comparison between welfare costs in the model and the literature.

3.1 Labor market flows

The economy is populated by a large number of identical households, normalized to one. Each household consists of a continuum of infinitely living agents. We consider a standard labor and matching model à la DMP. Employment N_t evolves according to

$$N_t = (1 - s)N_{t-1} + M_t \tag{1}$$

where s denotes the separation rate. M_t, the number of hirings per period, is determined by a constant-returns-to-scale matching function $M_t = \chi V_t^{\psi} S_t^{1-\psi}$, with $0 < \psi < 1$, $\chi > 0$

[17]We also depart from Garin (2015) as we do not introduce fixed training costs and time-varying vacancy, that are known to change *per se* the prediction of the basic DMP model (even without financial constraints).
[18]We present in section 5.4 an extension with capital.

a scale parameter measuring the efficiency of the matching function, and V_t the number of vacancies. Following Blanchard & Gali (2010), we suppose that a pool of jobless individuals, S_t, is available for hire at the beginning of period t. This implies that the pool of jobless agents is larger than the number of unemployed workers. Indeed, individuals are either employed or willing to work (full participation) at all times so that S_t is given by

$$S_t = U_{t-1} + sN_{t-1} = 1 - (1-s)N_{t-1} \tag{2}$$

where $U_t = 1 - N_t$ is the stock of unemployed workers when the size of the population is normalized to 1 and full participation is assumed. U_t thus measures the fraction of the population left without jobs after hiring takes place in period t. Among agents looking for jobs at the beginning of period t, a certain number M_t is hired, and they start working in the same period. Only workers in the unemployment pool S_t at the beginning of the period can be hired ($M_t \leq S_t$). The ratio of aggregate hires to the unemployment pool is the rate at which jobless people in the pool find a job, $\Psi_t \equiv M_t/S_t$, whereas $\Phi_t \equiv M_t/V_t$ is the rate at which vacancies are filled. Labor market tightness θ_t equals $\frac{V_t}{S_t}$.[19]

3.2 Households

Households maximize the utility function of consumption and labor. In each period, an agent can engage in only one of two activities, working or enjoying leisure. Employment lotteries ensure that agents are insured against the individual labor idiosyncratic risk. Hence, the representative household's preferences are

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \mu^t \{ N_t U^n \left(C_t^n\right) + (1-N_t) U^u \left(C_t^u + \Gamma\right) \} \right] \tag{3}$$

where $0 < \mu < 1$ is the discount factor and Γ the utility of leisure. C_t^z stands for the consumption of employed ($z = n$) and unemployed agents ($z = u$). We assume that $U(C_t^n) = \frac{(C_t^n)^{1-\sigma}}{1-\sigma} \equiv \tilde{U}_t^n$ for employed workers and $U(C_t^u + \Gamma) = \frac{(C_t^u+\Gamma)^{1-\sigma}}{1-\sigma} \equiv \tilde{U}_t^u$ for unemployed workers, with $\sigma > 0$ the coefficient of relative risk aversion. The utility function (3) has several interesting features. First, it is consistent with Rogerson (1988)'s lotteries.[20]

[19] The jobless rate is thus a convex function of the job-finding rate, $S = \frac{s}{s+(1-s)\Psi}$.

[20] Individuals can either work or not, which creates ex-post heterogeneity across agents. The assumption of a representative worker is made possible in this framework by using Rogerson (1988)'s lotteries because they provide full-insurance against the risk of not-working. Indeed, there is a contract between an insurance company and the household, which commits the household to work with probability N_t. The contract itself has been traded on a competitive market, so the household gets paid whether it works or not. Households'

Secondly, as in Greenwood et al. (1988), labor supply (and thus the Nash-bargained wage) are determined independently from intertemporal consumption-savings choices. Rather than being a drawback, this implication of the utility function has the advantage of isolating the effect studied here : the impact of financial frictions on wage bargaining and labor market fluctuations.[21]

The budget constraint is

$$[N_t C_t^n + (1 - N_t) C_t^u] + B_t \leq R_{t-1} B_{t-1} + N_t w_t + (1 - N_t) b_t + T_t \tag{4}$$

where w denotes real wage and b unemployment benefits. T is a lump-sum transfer from the government. Moreover, B represents private bond-financing firms and R is the gross investment return associated with these loans. Households' labor opportunities evolve as follows:

$$N_t = (1 - s) N_{t-1} + \Psi_t S_t \tag{5}$$

Each household chooses $\{C_t^n, C_t^u, B_t\}$ to maximize (3) subject to (4) and (5).[22]

3.3 Entrepreneurs and Firms

The economy includes many identical firms. Entrepreneurs maximize the following sum of expected utilities:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t^F\right) \right] \tag{6}$$

where β denotes the entrepreneurs' discount factor, and $\mu > \beta$, implying that workers are more patient than firms. Their budget constraint is

$$C_t^F + R_{t-1} B_{t-1} + q_t [L_t - L_{t-1}] + w_t N_t + \bar{\omega} V_t \leq Y_t + B_t \tag{7}$$

where B is private debt, L productive land or infrastructure, and q its price. Moreover, wN denotes total wages, with N the number of employees, whereas Y is the final output, and $\bar{\omega}$ the vacancy posting cost. Each firm has access to a Cobb-Douglas constant-return-to-scale

optimal decision imply that $U_t(C_t^n) = U(C_t^u + \Gamma)$, as can be seen from household's optimal consumption decision (equation (27) in Appendix B.1). Since all households are *ex ante* identical, all will choose the same contract. Although households are ex ante identical, they will differ *ex post* depending on the outcome of the lottery: a fraction N_t, of the continuum of households will work and the rest will not.

[21] Hansen (1985) and Jung & Kuester (2011) use alternative utility functions so that intertemporal consumption-savings choices affect labor decision and the Nash bargained wage. In our sensitivity analysis (section 5.4.2), we show that our results are robust with their specification.

[22] See Appendix B for a complete description of the model.

production technology combining workers and infrastructure (land):

$$Y_t = A_t L_{t-1}^{1-\alpha} N_t^{\alpha} \tag{8}$$

where $0 < \alpha < 1$ and A_t denotes total factor productivity in the economy, assumed to evolve stochastically as follows: $\log A_t = \rho_a \log A_{t-1} + (1 - \rho_a) \log A + \varepsilon_t^a$, with ε_t^a the iid innovations. Firms' activity can be financed by funds lent by households under imperfect debt contracts. Enforcement limits *à la* Kiyotaki & Moore (1997) imply that entrepreneurs are subject to collateral constraints. As Quadrini (2011) and Jermann & Quadrini (2012), we assume that, at the beginning of the period, firms can access financial markets to finance both their expenditures (i.e., consumption by entrepreneurs and land investments) as well as the costs associated with working capital within the period. Moreover, following Petrosky-Nadeau (2013) and Wasmer & Weil (2004), hiring costs are the working capital to finance. Entrepreneurs can thus borrow from agents subject to the following collateral constraint [23]:

$$B_t + \bar{\omega} V_t \leq m \mathbb{E}_t \left[q_{t+1} L_t \right] \tag{9}$$

This constraint shows that if the entrepreneur fails to repay the loan, the lender can seize the collateral. Given that liquidation is costly, the lender can recover up to a fraction, m, of the value of collateral assets. m is the exogenous loan-to-debt ratio. It is possible to show that, in presence of standard levels of uncertainty[24], firms are always collateral constrained (equation (9) holds with equality). Thus, the debt limit eventually determines the equilibrium level of corporate debt and workers savings. Vacancies are such that

$$N_t = (1 - s) N_{t-1} + \Phi_t V_t \tag{10}$$

Each entrepreneur chooses $\{C_t^F, L_t, B_t, V_t, N_t\}$ to maximize (6) subject to (7), (9), and (10), where Y_t is given by (8). From the first order conditions on N_t and V_t, we get the Job Creation (JC) curve is

$$\frac{(1 + \varphi_t) \bar{\omega}}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t + (1 - s) \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} \frac{(1 + \varphi_{t+1}) \bar{\omega}}{\Phi_{t+1}} \right] \tag{11}$$

where φ_t is the Lagrangian multiplier associated with the credit constraint (Equation (9)) and captures the "credit multiplier" of this model, and λ_t^F is the Lagrangian multiplier associated with the budget constraint of the entrepreneurs (Equation (7)). The credit multiplier (φ)

[23] In section 5.4, we will analyse the impact of alternative formulations of the debt limit.

[24] For a discussion see, among others, Iacoviello (2005). See Appendix B.2.

appears both on the LHS and RHS of Equation (11). Nevertheless, there is almost no persistence in the adjustment of φ_t; after a jump at the time of the shock, it comes back to its steady-state value. We thus shift our attention to its impact on the LHS of (11). In recession, tight credit conditions (large values of φ_t) drive up the opportunity costs associated with vacancy posting, $\bar{\omega}\,(1+\varphi_t)$. This introduces a countercyclical wedge that has the potential to magnify productivity shocks. If the real wage is sufficiently sluggish, the adjustments of quantities on the labor market can thus be large.

3.4 Wages

The wage is the solution of the maximization of the generalized Nash product $\max_{w_t}\left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right)^{\epsilon}\left(\frac{\mathcal{V}_t^H}{\lambda_t}\right)^{1-\epsilon}$, with $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}}$ the marginal value of a match for a firm and $\mathcal{V}_t^H = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}}$ the marginal household's surplus from an established employment relationship. ϵ denotes the firm's share of a job value, i.e., firms' bargaining power. The wage curve (WC) is

$$\begin{aligned} w_t \;=\; & \underbrace{\epsilon(b+\Gamma)}_{(a)} + \underbrace{(1-\epsilon)\frac{\partial Y_t}{\partial N_t}}_{(b)} \\ & \underbrace{+(1-\epsilon)\,(1-s)\mathbb{E}_t\left[\underbrace{(1+\varphi_{t+1})}_{(3)}\left\{\underbrace{\frac{\bar{\omega}}{\Phi_{t+1}}\left(\beta\frac{\lambda_{t+1}^F}{\lambda_t^F}-\mu\frac{\lambda_{t+1}}{\lambda_t}\right)}_{(1)}+\underbrace{\left(\mu\frac{\lambda_{t+1}}{\lambda_t}\right)\bar{\omega}\theta_{t+1}}_{(2)}\right\}\right]}_{(\Sigma)} \end{aligned} \tag{12}$$

where (a) represents the weight of the reservation wage in total wage and $(b)+(\Sigma)$ is the workers' gain from the match. This gain can be decomposed into the marginal productivity of the new employed worker, (b), and the saving on search costs if the job is not destroyed next period (Σ). Notice that when firms and workers have the same discount factor ($\mu=\beta$, $\varphi=0$, thus $\lambda_t=\lambda_t^F$), equation (12) collapses to the standard Blanchard & Gali (2010) wage curve:

$$w_t \;=\; \epsilon(b+\Gamma)+(1-\epsilon)\frac{\partial Y_t}{\partial N_t}+(1-\epsilon)(1-s)\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\bar{\omega}\theta_{t+1}\right] \tag{13}$$

Without financial frictions, flexible wages can make the job surplus insensitive to shocks. Indeed, an increase in labor productivity $\frac{\partial Y_t}{\partial N_t}$ encourages firms to create vacancies. The resulting increase in the job-finding rate puts upward pressure on wages (through the rise in labor market tightness θ in equation (13)). Workers also get a share of the rise in labor productivity in the wage bargaining process. Procyclical wages then virtually soak up hiring incentives in booms.

The comparison of (12) with (13) shows that financial frictions enter the wage equation through term (Σ) and affect wage bargaining directly through the "credit multiplier" φ (term (3)), and indirectly via the gap between agents' discount rates (term (1)). The "credit channel" (term (3)) is also at work in Petrosky-Nadeau (2013) and Wasmer & Weil (2004) and discussed in Monacelli et al. (2011). In booms, financial conditions improve (φ goes down), which reduces workers' share of the job surplus. Thus, the credit multiplier tends to strengthen the firm's bargaining position. The original feature of our model lies in the impatience gap between the firm and the worker (Term (1)). This impatience gap is proportional to the expected time the firm has to wait before filling a vacancy ($1/\Phi_{t+1}$) and thus to the average quantity of credit required by firms to finance their hiring policy. In booms, due to congestion externalities, the firm faces recruiting difficulties and the expected duration of credit (used to finance hiring costs) expands. Without impatience gap, agents perceive symmetrically the credit costs induced by the financial constraint: in this case, term (1) cancels out. In contrast, because of the impatience gap, the more impatient the entrepreneurs (the future matters less for entrepreneurs than for workers), the lower their assessment of the value of filled jobs, and thus the match surplus. As a result, entrepreneurs are reluctant to accept wage increases in booms. This introduces an original counter-cyclical component in the wage equation. Therefore, financial frictions *à la* Kiyotaki & Moore (1997) (and more precisely, the impatience gap between workers and entrepreneurs) introduce a wage moderation component that makes the match surplus more responsive to shocks. In section 4 we will discuss under which conditions this mechanism can produce sizeable fluctuations in the labor market tightness and the job finding rate so as to solve the Shimer puzzle.

3.5 Markets clearing

In order to close the model, we assume that the government does not accumulate debt and pays unemployment benefits using a lump-sum tax; i.e., $T_t = -(1 - N_t)b_t$. The private-debt market clears. Good market equilibrium requires the condition $Y_t = N_t C_t^n + (1 - N_t)C_t^u + \bar{\omega}V_t + C_t^F$. Finally, we assume that land supply is fixed and that the land market clears in each period; i.e., $L_t = 1$.

4 Large welfare costs of fluctuations *and* volatile labor market: a non-trivial coincidence

In order to provide intuitions for the mechanisms at work, we examine in this section a simplified version of the model: workers are assumed to be risk neutral. We first deduce that welfare costs of the business cycle are reduced to production losses, i.e. the gap between deterministic steady-state unemployment and the average unemployment of the stochastic economy. Secondly the financial frictions must then be captured by an ad-hoc function linking financial imperfections to an exogenous labor productivity shock. We look at the model's equations at the steady state and compare its deterministic unemployment level (in the stabilized economy) and to its predictions on average unemployment (in the fluctuating economy around the steady state).

Let us denote y the exogenous labor productivity shock. The labor market equilibrium is defined by the combination of the Beveridge curve (BC), the job creation (JC) and the wage curve (WC) at the steady state:[25]

$$(BC) \qquad u(\theta(y)) = \frac{s}{s + \Psi(\theta(y))} \tag{14}$$

$$(JC) \qquad \frac{\omega(\varphi(y))}{\Phi(y)} = \frac{y - w(\theta(y), \varphi(y), y)}{1 - (1-s)\,\beta} \tag{15}$$

$$(WC) \qquad w(\theta(y), \varphi(y), y) = \epsilon(b + \Gamma) + (1-\epsilon)\,(y + \Sigma(\theta(y), \varphi(y), y)) \tag{16}$$

where $\omega(\varphi(y)) = \bar{\omega}\,(1 + \varphi(y))$ and

$$\Sigma(\theta(y), \varphi(y), y) = (1-s)\beta\omega(\varphi(y))\theta(y) + (1-s)\beta\frac{\varphi(y)}{1-\varphi(y)}\omega(\varphi(y))\left(\theta(y) - \frac{1}{\Phi(\theta(y))}\right) \tag{17}$$

$$\Psi(\theta(y)) = \chi\theta(y)^{\psi}, \quad \text{equivalently} \quad \Phi(\theta(y)) = \chi\theta(y)^{\psi-1} \tag{18}$$

$$\varphi(y) = 1 - \frac{\beta}{\mu}y^{\zeta} \tag{19}$$

Equation (17) captures the savings on search costs, Σ, if the job is not destroyed next period. Σ can be decomposed into two parts: i) the standard component proportional to $\theta(y)$ – that summarizes recruiting difficulties (i.e. the last term on RHS of the standard DMP wage equation (13), which is also the first term of the RHS of equation (17)) and ii) an original component that captures the interactions between financial and labor market

[25]The Beveridge curve (BC) is derived from labor market flows, equation (1); the job creation (JC) comes from equation (11) and the wage curve (WC) is equation (12) in which we use the steady-state relationship between discount rates in first-order conditions (28) and (32) in Appendix B.

frictions (i.e., respectively $\varphi(y)$ and $\theta(y)$ that are in the second part of the RHS of equation (17)). Equation (18) is the matching function written equivalently for the job finding rate Ψ or the probability to fill a vacancy Φ. In equation (19), we approximate the impact of financial frictions on the labor market with $\varphi(y)$ where $\zeta > 0$. Consistently with our DSGE model, we assume $\varphi'(y) < 0$, i.e., during booms, the cost of credit falls.[26] When $\zeta = 0$ and $\beta = \mu$, there are no financial frictions ($\varphi(y) = 0$) and we recover the standard DMP model as a special case of our simplified model. We provide in this section economic intuitions. Formal demonstrations are reported in Appendix C.

4.1 The dynamics of the Beveridge curve

With risk neutral agents, welfare costs of fluctuations are captured by the gap between the unconditional expectation of the unemployment rate $\mathbb{E}_0 u$ and its deterministic counterpart $\overline{u}$. Unemployment dynamics are determined by the Beveridge Curve (BC). Equation (14) at the steady state yields the deterministic unemployment level:

$$\overline{u} = \frac{s}{s + \bar{\Psi}} \tag{20}$$

where $\bar{x}$ denotes the steady-state value of variable x. Equation (20) underlines that unemployment is a convex function of the job finding rate Ψ. This non-linearity implies that in an economy with a fluctuating job finding rate (hereafter JFR), unemployment rises sharply in recession while it falls moderately in expansion. As a result, average unemployment (hence average consumption) is higher (is lower) than stabilized unemployment (stabilized consumption), thereby generating welfare costs of fluctuations. A simple way of deriving this unemployment gap consists in computing a second-order Taylor expansion of (14) around the steady state, we get

$$\mathbb{E}_0 u - \overline{u} \approx -\frac{s}{(s + \bar{\Psi})^2}(\mathbb{E}_0 \Psi - \bar{\Psi}) + \frac{s}{(s + \bar{\Psi})^3}\mathbb{E}_0(\Psi - \bar{\Psi})^2 \tag{21}$$

Therefore, the gap between average unemployment ($\mathbb{E}_0 u$) and its deterministic steady state ($\bar{u}$) depends *(i)* on the gap between average JFR ($\mathbb{E}_0 \Psi$) and its deterministic steady state ($\bar{\Psi}$), and *(ii)* on JFR volatility $\mathbb{E}_0(\Psi - \bar{\Psi})^2$. Taking a second-order Taylor expansion of Ψ by using the matching function (18) yields

$$\mathbb{E}_0 \Psi - \bar{\Psi} \approx \frac{1}{2}(\Psi''\theta' + \Psi'\theta'')\mathbb{E}_0(y - \bar{y})^2 \quad \text{and} \quad \sigma^2_\Psi \equiv \mathbb{E}_0(\Psi - \bar{\Psi})^2 = (\Psi'\theta')^2\sigma^2_y \tag{22}$$

[26] We also assume that the upper bound of the y-distribution is lower than $(\mu/\beta)^{1/\zeta}$, so that $\varphi(y) > 0$.

where $\mathbb{E}(y) = \overline{y} = 1$.[27] Equation (21) becomes

$$\mathbb{E}_0 u - \overline{u} \approx \underbrace{-\frac{s}{(s+\bar{\Psi})^2}\frac{1}{2}(\underbrace{\Psi''\theta'}_{G_1} + \underbrace{\Psi'\theta''}_{G_2})\sigma_y^2}_{\text{Curvature of hiring decisions}} + \underbrace{\frac{s}{(s+\bar{\Psi})^3}(\Psi'\theta')^2\sigma_y^2}_{\text{Curvature of }(BC)} \tag{23}$$

Notice that, because of the matching function (18), JFR is increasing ($\Psi' > 0$) and concave ($\Psi'' < 0$). In addition, in booms, labor market tightness increases ($\theta' > 0$) while the sign of θ'' is not a priori known. As a result, the first and third terms in equation (23) tend to increase welfare costs while the impact of middle term G_2 depends on the sign of θ''.

Equation (23) has several interesting features. First, equation (23) illustrates the importance of taking into account non-linearity in the Beveridge curve (Curvature of (BC)) to produce potentially sizeable welfare costs of fluctuations:[28] Only by taking into account non-linear dynamics between unemployment and JFR, do we get a gap between average and deterministic unemployment due to asymmetries in unemployment dynamics. Indeed, during booms, the increase in JFR is partly offset by the decrease in unemployment. In recessions, the fall in JFR is amplified by the increase in unemployment. The fall in JFR in recessions is then larger than the JFR rise in expansions. Hence, average JFR is lower than its deterministic counterpart ($\mathbb{E}_0\Psi - \bar{\Psi} < 0$). Then, in equation (21), average unemployment lies above its steady-state value. This gap is also magnified by JFR volatility (σ_Ψ^2).

Second, in the economic mechanisms described above, we look at the gap between average JFR ($\mathbb{E}_0\Psi$) and its deterministic counterpart ($\bar{\Psi}$), which, in equation (22), depends on the concavity of the matching function (Ψ' and Ψ''), the responsiveness of labor market tightness to aggregate shocks θ' and the curvature of labor market tightness θ''. Notice also that equation (23) shows why large JFR fluctuations (and thus the necessity to solve the Shimer puzzle) are a key element to obtain sizeable welfare costs of business cycles. Indeed, a large θ' (that appears twice in equation (23)) widens the gap $\mathbb{E}_0 u - \overline{u}$.

Finally, financial frictions enter the picture through their effect on fluctuations of labor market tightness θ. In particular, financial frictions affect welfare costs through two channels: *(i)* the size of JFR fluctuations, which depends on θ', and *(ii)* the curvature of labor market

[27] For the sake of brevity, we denote as Ψ' the first derivative of JFR Ψ with respect to labor market tightness θ around the steady state $\overline{\theta}$, and θ' the first derivative of labor market tightness with respect to exogenous labor market productivity y around the steady state $\overline{y}$. The same notations apply to second derivatives.

[28] A first-order Taylor expansion of the JFR equation would yield $\Psi \approx \overline{\Psi} + \Psi'\theta'(y - \overline{y})$. Taking expectations of this expression, we get $\mathbb{E}_0\Psi - \overline{\Psi} \approx 0$ as $\mathbb{E}_0 y = \overline{y}$. As a result, (21) would imply, in a first-order expansion, $\mathbb{E}_0 u - \overline{u} \approx 0$. Welfare costs of fluctuations are zero as falls in unemployment in booms exactly compensates the rise in unemployment in recessions.

tightness, which determines the sign of θ''. In what follows we analyze the role of both component.

4.2 Shimer puzzle, financial frictions and the size of job finding rate fluctuations

In this section we examine the response of labor market tightness to productivity shocks (i.e., θ' in equation (23)), in a model with and without financial frictions (hereafter FF). It is straightforward to see in (23) that the larger θ', the greater the response of JFR to aggregate shocks, the more sizable welfare costs of fluctuations and the closer the model to the data (hence, solving Shimer (2005)'s puzzle).

Proposition 1. *Financial frictions (FF) can lead to larger fluctuations of labor market tightness, i.e.* $\theta'|_{FF} > \theta'|_{noFF}$*: the response of labor market tightness to an exogenous productivity shock can be higher in a model with FF with respect to a standard DMP model.*

Proof. See Appendix C.1 for a derivation of parameter restrictions such that proposition 1 holds. □

Shimer (2005) points out that the standard DMP model fails to generate realistic JFR fluctuations (i.e. $\theta'|_{noFF}$ is too low). Indeed, following a positive productivity shock, firms are enticed to post more vacancies, which raises labor market tightness. However, higher wages (higher labor productivity pushes wage upward in equation (13)) absorb most of the productivity increase (higher w in equation (15)), eliminating the incentive for vacancy creation. The pro-cyclical wage response tends to dampen firms' hiring incentives in booms, which generates low JFR volatility, hence low welfare costs of fluctuations.[29]

Proposition 1 shows that the financial frictions can magnify the impact of the business cycle on JFR volatility ($\theta'|_{FF} > \theta'|_{noFF}$), without assuming ad-hoc wage rigidity. This result is not trivial. Indeed, one the one hand, FF have a direct impact on hiring costs for each firm: in boom, the credit cost φ is reduced, and so are hiring costs (φ goes down on the LHS of equation (15)). This force magnifies the impact of productivity shocks on labor market tightness (the "credit multiplier" effect). In addition, the endogenous wage dynamics must not be too pro-cyclical so as to make the decline in the hiring costs in booms a dominant driver of job creation (the increase in w must be limited in equation (15). This is the

[29] Assuming a limited wage response to the business cycle (with low workers' low bargaining power as in Hagendorn & Manovskii (2008) or wage rigidity as initially suggested by Hall (2005*b*)) helps the model generate data-consistent JFR volatility.

economic interpretation of parameter restrictions derived in Appendix C.1). This is made possible through FF. Indeed, FF actually introduce new components in (WC), which induces an endogenous wage rigidity: the impatience gap introduces a countercyclical component in wages. In booms, the average duration to fill a vacancy goes up[30], which increases the quantity of credit the firm needs to borrow but the surplus from the match that will be used to finance these credit costs is perceived to be lower by the impatient entrepreneur than by the (patient) worker.[31] This makes the entrepreneur more reluctant to accept wage increases in booms. This counter-cyclical component in the wage setting rule is the channel through which our model can generate an endogenous wage rigidity, leading to $\theta'|_{FF} > \theta'|_{noFF}$. This channel is specific to Kiyotaki & Moore (1997)'s model.

Under parameter restrictions, FF can then increase the sensitivity of the JFR to productivity shocks y, leading to a larger gap $\mathbb{E}_0 u - \overline{u}$ in a economy with FF than without FF. A byproduct of this result is that FF can increase the sensitivity of the labor market tightness to the business cycle, and solve the Shimer puzzle. Note that the model must reproduce not only labor market fluctuations but also match volatility of financial variables, which we did not analyze here. This will be done in section 5.

4.3 The concavity of labor market tightness with respect to aggregate shocks

We now shift our attention to term G_2 in equation (23), which incorporates θ''. Given our matching function, equation (18), JFR is a positive function of labor market tightness, $\Psi' > 0$. Therefore, this term increases welfare costs of fluctuations only if $\theta'' < 0$.

Proposition 2. *Without financial frictions, the model only exhibits a θ-convexity (i.e. $\theta'' > 0$), whereas, with financial frictions, the model can exhibit a θ-concavity (i.e. $\theta'' < 0$).*

Proof. See Appendix C.2 for a derivation of parameter restrictions such that proposition 2 holds. □

The economic intuition of Proposition 2 relies on the job creation condition JC, equation (15). The latter relates vacancy posting costs on the LHS to labor market tightness θ on the RHS. Two opposite forces are at work in response to a productivity shock.

[30]This is measured by the decline in the probability to fill a vacancy Φ due to strong congestion effects for firms, and thus the increase in the average cost of recruiting $\frac{\bar{\omega}}{\Phi}$.

[31]In equation (17), the last term on the RHS, $-\frac{\omega}{\Phi}\varphi$, captures the wage reduction induced by the smaller evaluation of the match surplus by the impatient entrepreneur). This term is absent in the standard DMP wage equation.

1. During booms, as unemployment falls, hiring takes more time so that firms create more vacancies to reach their targeted employment level. To the contrary, during recessions, hiring is not difficult because the unemployment rate is large. Thus, a slight fall in vacancies is enough to meet the job creation condition. Labor market tightness hence increases more in expansion than it falls in recession. Indeed, given that the matching function exhibits decreasing marginal returns to vacancies, the job creation condition JC is satisfied for greater variations in job creation during booms than during recessions.

   Without financial frictions, these mechanisms unambiguously lead the hiring decision rule –i.e., θ – to be a convex function of productivity shocks, $\theta'' > 0$. Thus, average labor market tightness is greater than its deterministic steady-state value. This reduces the gap $\mathbb{E}_0 u - \overline{u}$, thereby dampening welfare costs of the business cycle. Notice that this latter effect on θ'' was emphasized by Hairault et al. (2010) in a standard DMP model. This result also holds in a model without financial frictions and with rigid real wages.

2. Financial frictions introduce a new component into the wage equation. In booms, the large increases in the labor market tightness also signal a rise in the credit cost – included in the bargained wage because entrepreneurs perceive less future profits than workers. Given that the credit duration[32] increases more in booms than it declines in recessions, smaller variations of the labor market tightness are needed in booms than in recessions to satisfy the job creation condition. This mechanism can lead labor market tightness to be a concave function of productivity ($\theta'' < 0$).

The two forces work in opposite directions in the fluctuating economy: the first effect increases average JFR while the second one lowers it. Notice that the second effect is specific to financial frictions and constitute an additional channel widening the gap between average unemployment and its value at the deterministic steady state – and therefore, enlarging the welfare costs of fluctuations. In Appendix C.2, we derive a threshold degree for financial frictions to produce θ-concavity. We show that when financial frictions are strong enough, they increase welfare costs of fluctuations. They are thus at the heart of the large magnitude of business cycle costs in our model.

Notice finally that the second force, that generates θ-concavity, is due to the endogenous wage-adjustment mechanism. Therefore, the extreme assumption of exogenous rigid wages cannot generate this θ-concavity in a model with financial frictions. In fact, under rigid

[32]These costs are directly linked to the increase in the time duration to fill a vacancy $\frac{1}{\Phi}$.

wages, financial frictions magnify the impact of productivity shocks on the surplus from the match. This then leads to more convex decision rules on θ. Notice also that, for both large degrees of financial imperfections and exogenous rigid wages, the resulting θ-convexity can even induce welfare gains: in response to positive productivity shocks, firms enjoy both an increase in productivity and a fall in credit costs. As wages do not increase in expansion, firms may benefit so much from booms that average unemployment can lie below its deterministic value, thereby generating welfare gains of fluctuations. This point is analytically derived in Appendix C.2 and illustrated in section 5.4.1.

5 Quantitative analysis

Our theoretical analysis does not yield any prediction on the volatility of financial variables. We then need a calibrated full DSGE model to assess this point. We also need the DSGE model to see whether parameter restrictions found in the simplified model hold or not in a realistic model of the US economy. For our quantitative calculation of business-cycle costs to be relevant, we need to match the volatility of data. Overcoming the "Shimer puzzle" is thus a necessary condition for our exercise. Finally, we will use the full DSGE model for the sensitivity analysis.

5.1 Calibration

Preference, technology and shocks. The calibration is based on quarterly US data. The discount factor for patient agents is consistent with a 4% annual real interest rate. For the impatient consumer, we set $\beta = 0.99$, which is within the range of values chosen by Iacoviello (2005)[33]. The risk aversion is set to 1 for firms and 2 for workers. Both values lie within a standard interval in the literature. In addition, the firm is characterized by a lower risk aversion because, as shown by Iacoviello (2005), such a calibration ensures that the borrowing constraint is binding for a wide range of volatility shocks, impatience levels, and loan-to-value ratios (m values). The technological shock is calibrated as in Hairault et al. (2010). We choose the standard deviation of technological shock to reproduce the observed GDP standard deviation.

Financial frictions. The corporate debt-to-GDP ratio pins down the value of m in the collateral constraint. To this end, we use the average corporate debt over GDP for 2001-

[33]Notice that an extremely low degree of impatience heterogeneity is sufficient for debt limits to hold.

2009 (debt outstanding, annual data, corporate sector, Flow of Funds Accounts tables of the Federal Reserve Board, see Appendix A).

Labor market. Employment level N is consistent with the average unemployment rate ($N = 0.88$) estimates of Hall (2005*a*).[34] As in Shimer (2005), the quarterly separation rate s is 0.10, so jobs last for about 2.5 years on average. Using steady-state labor-market flows, we infer Ψ given s and N. This leads to $\Psi = 0.423$. This value is lower than in the usual DMP model. Indeed, the pool of job seekers is larger in Blanchard & Gali (2010) than in the standard DMP model. The elasticity of the matching function with respect to the number of job seekers is $\psi = 0.5$, which lies within the range estimated in Petrongolo & Pissarides (2001). The efficiency of matching, χ, is set such that firms with a vacancy find a worker within a quarter with a 95% probability, which is consistent with Andolfatto (1996). The cost of posting a vacancy, $\bar{\omega}$, is set to 0.17 as in Barron & Bishop (1985) and Barron et al. (1997). We obtain $\frac{\bar{\omega}V}{Y} = 0.0179$, which is within the range found in the literature (0.005 in Chéron & Langot (2004) or 0.05 in Krause & Lubik (2007)). The utility of leisure parameter, Γ, is pinned down so as to match $b = 0.7$ as in Hall & Milgrom (2008). We obtain $\Gamma = 0.18$, leading to $b + \Gamma = 0.88$, a lower value than one used by Hagendorn & Manovskii (2008) ($b = 0.95$ with $\Gamma = 0$). It is also lower than the parameters combination allowing Hall & Milgrom (2008) to control for the unemployment rate, namely the sum of leisure value ($b = 0.71$ with $\Gamma = 0$) and the "employer's cost of delay" ($\varsigma = 0.27$) which leads to $b + \varsigma = 0.98$. Table 1 summarizes the calibration.

5.2 Matching business-cycle volatility

In this section, we document the unconditional business cycles facts on financial variables and labor market adjustments. Our contribution lies also in bringing together financial data (from Jermann & Quadrini (2012)) and data from the labor market literature (Shimer (2012)). In both markets, we focus on fluctuations in quantities (debt, unemployment) as well as equilibrium prices (interest rate, wage).[35] Table 2, column (1), reports business-cycle

[34] According to Hall (2005*a*), the observed high transition rate from "out of the labor force" directly to employment suggests that a fraction of those classified as out of the labor force are nonetheless effectively job-seekers. Hall (2005*a*) adjusts the US unemployment rate to include individuals out of the labor force who are actually looking for a job.

[35] All data have been recomputed and updated so that our sample covers five recession episodes from 1976 January through January 2013 (see Appendix A for a complete description of the data set). Previous works that study the interaction between financial and real variables in DSGE models such as Monacelli et al. (2011) and Christiano et al. (2010) summarize labor market adjustments using only fluctuations in employment and unemployment.

Table 1: Calibration

(a) External information			
Notation	Label	value	Reference
β	discount factor (impatient)	0.99	Iacoviello (2005)
α	production function	0.99	Iacoviello (2005)
σ_W	risk aversion, worker	2	
σ_F	risk aversion, firm	1	Iacoviello (2005)
s	Job separation rate	0.1	Shimer (2005)
N	Employment	0.88	Hall (2005*a*)
ψ	Elasticity of the matching function	0.5	Petrongolo & Pissarides (2001)
$\bar{\omega}$	cost of job posting	0.17	Barron et al. (1997) and Barron & Bishop (1985)
$\frac{b}{w}$	replacement ratio	0.7	Hall & Milgrom (2008)
$\bar{A}$	average TFP	1	Normalization
ρ_A	Persistence	0.95	Hairault et al. (2010)
(b) Empirical target			
Notation	Label	value	Empirical target
μ	discount factor (patient)	$1/(1.04^{\frac{1}{4}})$	Annual real rate of 0.04
χ	scale parameter of matching function	0.634	Probability of filling a vacancy $\Phi = 0.95$
m	collateral constraint	0.61	corporate debt to GDP ratio $B/Y = 0.595$
σ_A	Standard deviation	0.0031	Observed σ_Y
(c) Derived parameter values			
Notation	Label	value	
Ψ	Job finding rate	0.423	
Γ	preference	0.18	

properties found in the data.

The volatility of real wages is not close to zero in the data. Moreover, it is larger than that of labor productivity. This clearly suggests that real wage rigidity (implying a zero standard deviation for fluctuations in w) is not a realistic explanation for the strong cyclicality of labor market aggregates. The correlation between unemployment and vacancies $corr(U_t, V_t)$ summarizes the dynamics around the Beveridge curve. The negative covariance is consistent with the view that aggregate shocks are more important than reallocation shocks at business-cycle frequency. As expected, the correlation between unemployment and job finding rate $corr(U_t, \Psi_t)$ is negative. Jung & Kuester (2011) point out that mean unemployment exceeds steady-state unemployment when the job-finding rate and the unemployment rate are non-positively correlated and the average job-finding rate is lower than the steady-state job-finding rate.[36] This is the case in our model. Comparing columns (1) and (2) of Table

[36] This can be inferred from the employment-flow equation taken at the steady state $sN_t = \Psi U_t$ where $N_t = 1 - U_t$. Hence, $s\mathbb{E}(1_t - U_t) = cov(U_t, \Psi_t) + \mathbb{E}(U_t)\mathbb{E}(\Psi_t)$. Subtracting the steady state from both sides of the above equation, leads to $\mathbb{E}(U_t) - u_t = -\frac{1}{s+\Psi}\left[cov(U_t, \Psi_t) + (\mathbb{E}(\Psi_t) - \Psi_t)\,\mathbb{E}(U_t)\right]$. We deduce that if (i) $\mathbb{E}(\Psi_t) - \Psi_t < 0$ and (ii) $cov(U_t, \Psi_t) < 0$, then necessarily, $\mathbb{E}(U_t) - u_t > 0$. The correlation at the bottom of

Table 2: Business-cycle volatility : Models versus data

	(1) Data std(.)		(2) Benchmark Model std(.)		(3) Without Financial Frictions std(.)	
Y	1.44	**	1.44	**	1.44	**
C	0.81	*	0.88	*	0.94	*
N	0.72	*	0.74	*	0.46	*
Y/N	0.54	*	0.28	*	0.56	*
w	0.62	*	0.49	*	0.49	*
U	7.90	*	5.45	*	3.71	*
Ψ	5.46	*	6.26	*	2.79	*
V	9.96	*	12.7	*	4.60	*
B	1.68	*	1.35	*		
q	3.21	*	2.59	*		
$corr(U, \Psi)$	-0.91		-0.86		-0.92	
$corr(U, V)$	-0.97		-0.71		-0.76	

** std (in percentage); * relative to GDP std
Colum (1) US Data, see Appendix A
$\sigma_A = 0.0031$ in column (2); $\sigma_A = 0.00615$ in column (3)

2, we note that the model generates volatile employment, vacancies, unemployment, and job-finding rates. The simulated volatilities of vacancy and job finding rate are even a bit higher than the observed volatilities, as we tend to slightly underestimate wage volatility. Interestingly, the predicted volatile adjustments on the labor market are not obtained under unrealistic fluctuations on financial markets. The model also reproduces fluctuations of corporate debt, nearly as volatile as in the data. Similar conclusions apply to the dynamics of land prices q. When the model is simulated without financial frictions (Table 2, column (3) and with a TFP process adjusted to match the volatility of output, the standard deviation of the job-finding rate relative to GDP is twice lower. Moreover, the relative standard deviation of the wage is the same as in the model with financial frictions, whereas the volatility of the outside option (given by $\widehat{\theta} = \frac{1}{\psi}\widehat{\Psi} > \widehat{\Psi}$) is more than twice as small. This clearly confirms that financial frictions dampen large movements in workers' outside options in the Nash bargained wage rule.

Matching volatility on the labor market: main mechanisms at work. Quantitative results suggest that parameter restrictions in Appendix C.1 hold in the full-fledged model. Economic intuitions have been discussed in section 4.

Table 2 suggests that *ii)* holds in the data.

Matching volatility on the financial market : main mechanisms at work. The model is able to replicate the volatility on the debt market. First, as the quantity of land does not adjust over the business cycle (which is a realistic assumption), any change in land demand results in a change in the real estate price q. Our model is then able to match the volatility of real-estate prices q.

It is noticeable that the model also reproduces a realistic volatility of corporate debt, B. To explain the intuition, let us consider a log-linearized version of the binding borrowing constraint, equation (9). The variance of the price of collateral is equal to the sum of the variance of corporate debt, vacancies and the covariance between debt and vacancies. In a model without interactions between labor and financial frictions, vacancies would not appear in the borrowing constraint. In this case, the variance of the price of collateral would equal the variance of corporate debt. Thus, the model would predict fluctuations in corporate debt as large as real-estate price q – which is counterfactual. In contrast, in our model, because of vacancies in the borrowing constraint we can predict a large variance of the real-estate price as well as a lower variance of corporate debt. This is due to the impact of the large variance of vacancies and the positive covariance between debt and vacancies. Hence, the interaction between labor and financial frictions also makes the model consistent with the observed volatility of financial variables.

5.3 Welfare cost of fluctuations

Decomposing the welfare cost of fluctuations. The expected lifetime utility of a worker is $\tilde{\mathcal{W}}^w = \mathbb{E}_0 \sum_{t=0}^{\infty} \mu^t U\left(C_t + (1-N_t)\Gamma\right)$, because $C_t \equiv N_t C_t^n + (1-N_t) C_t^u$ and the FOC on consumption imply $C_t^n = C_t^u + \Gamma$. We define welfare costs of business cycles, τ, as the fraction of steady-state consumption that workers would give up to be indifferent between the steady-state and the fluctuating economy. Welfare costs of fluctuations τ are such that $\sum_{t=0}^{\infty} \mu^t U\left([\bar{C} + (1-\bar{N})\Gamma](1-\tau)\right) = \tilde{\mathcal{W}}^w$, where variables marked with an overbar denote their steady-state values.[37] We deduce

$$\tau = 1 - \left[\tilde{\mathcal{W}}^w \frac{(1-\mu)(1-\sigma)}{(\bar{C} + (1-\bar{N})\Gamma)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

[37] A similar computation is possible for the firm's owner: $\sum_{t=0}^{\infty} \beta^t U\left(\bar{C}^F(1-\tau^F)\right) = \tilde{\mathcal{W}}^F = \sum_{t=0}^{\infty} \beta^t U\left(C_t^F\right)$. When introducing financial frictions, firms' welfare costs of fluctuations can also be taken into account. In this case, aggregate welfare costs would then be greater. We choose to focus on workers' welfare costs only to compare our results to the existing literature.

The result is reported in Table 3, line 1, column A. The business-cycle cost of fluctuations with financial frictions is 2.50% of workers' permanent consumption. This number is far larger than the estimates found by Lucas (1987, 2003), who reports a welfare cost of $\tau = 0.05\%$ with log utility. Notably, welfare costs are large even though agents can save by lending to firms; workers can actually smooth business cycles with savings.

One way to understand these quantitative results is to decompose the welfare cost into a "level effect" and a "business-cycle effect". The former component is due to the gap between the average level of consumption in the fluctuating economy $\mathbb{E}_0 C$ and its counterpart in the stabilized steady-state economy $\overline{C}$. The latter, the "business-cycle effect", entails costs that are directly derived from volatility in the economy, as in Lucas (1987, 2003). We use a second-order Taylor expansion of the utility function in the volatile economy. The crucial point at this stage is to consider the Taylor expansion around the mean of the stochastic economy, and not around the deterministic steady state. This ensures that the computation takes into account the gap between the mean (i.e., the stochastic steady state) and the deterministic steady state. We approximate welfare in the volatile economy as the sum of a consumption gap and Lucas' original measure. Indeed, we have

$$\tilde{\mathcal{W}}^w \approx \frac{1}{1-\mu} U\left(\mathbb{E}_0[C+(1-N)\Gamma]\right)\left[1-\frac{1}{2}\sigma(1-\sigma)\left(\gamma_c Var(\widehat{c})+\gamma_u Var(\widehat{u})+\gamma_{cu} Cov(\widehat{c},\widehat{u})\right)\right]$$

where we denote[38] $\widehat{x} = \frac{X_t - \mathbb{E}_0[X]}{\mathbb{E}_0[X]}$, for $x = C, U$ and $\gamma_c = \frac{\mathbb{E}_0[C^2]}{\mathbb{E}_0[(C+(1-N)\Gamma)^2]}$, $\gamma_u = \frac{\Gamma^2\mathbb{E}_0[(1-N)^2]}{\mathbb{E}_0[(C+(1-N)\Gamma)^2]}$ and $\gamma_{cu} = \frac{2\Gamma\mathbb{E}_0[C(1-N)]}{\mathbb{E}_0[(C+(1-N)\Gamma)^2]}$. This leads to

$$1-\tau \approx \underbrace{\left(\frac{\mathbb{E}_0[C+(1-N)\Gamma]}{\bar{C}+(1-\bar{N})\Gamma}\right)}_{=1-\tau_{Gap}}\underbrace{\left[1-\frac{1}{2}\sigma(1-\sigma)\left(\gamma_c Var(\widehat{c})+\gamma_u Var(\widehat{u})+\gamma_{cu} Cov(\widehat{c},\widehat{u})\right)\right]^{\frac{1}{1-\sigma}}}_{=1-\tau_{BC}} \tag{24}$$

τ_{BC} denotes the welfare costs of business cycles computed in the spirit of Lucas (1987, 2003) while τ_{Gap} denotes the welfare costs of the business cycle linked to the consumption gap between average and steady-state consumption. As consumption and labor are not separable in the utility function (equation (3)), unemployment also matters in τ. In addition, $(1-\tau) = (1-\tau_{BC})(1-\tau_{Gap})$ so that $\tau \approx \tau_{BC} + \tau_{Gap}$.

Welfare costs due to business-cycle fluctuations alone, in the spirit of Lucas' measure, τ_{BC}, are reported in Table 3, line 3, column A, and equal 0.27% of permanent consumption for

[38] Indeed, given that $\frac{\partial U}{\partial C} = U'$, $\frac{\partial U}{\partial N} = -\Gamma U'$, $\frac{\partial^2 U}{\partial C^2} = U''$, $\frac{\partial^2 U}{\partial N^2} = \Gamma^2 U''$ and $\frac{\partial^2 U}{\partial C \partial N} = -\Gamma U''$, we obtain, with the usual functional form $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$, $U' = x^{-\sigma}$ and $U'' = -\sigma x^{-\sigma-1} = -\sigma(1-\sigma)\frac{x^{1-\sigma}}{1-\sigma}\frac{1}{x^2}$.

Table 3: Decomposition of welfare costs of business cycle

	Worker with financial frictions A	Worker without financial frictions B
Total welfare cost		
1. $\tau \times 100$	2.50	0.12
Decomposing the welfare cost		
2. $\tau_{Gap} \times 100$	2.23	0.09
3. $\tau_{BC} \times 100$	0.27	0.03

line 1 = line 2 + line 3. $\sigma_A = 0.0031$ in all columns.

workers. They are five times as large as in Lucas (1987, 2003) (0.05%). This first result comes from labor-market fluctuations (which are magnified by financial frictions). The latter are neglected by Lucas. In our model, τ_{BC} is a function of not only $Var(\widehat{c})$, as in Lucas, but also $Var(\widehat{u})$ and $Cov(\widehat{c}, \widehat{u})$, which, for a given "level effect", clearly magnify the costs of cycles. The most striking result is the measure of $\tau_{Gap} = 2.23\%$ in Table 3, line 2, column A. It accounts for the great increase in business-cycle costs: 90% of welfare costs come from this consumption gap. In fact, in Lucas (1987, 2003), $\tau_{Gap} = 0$: average consumption coincides with steady-state consumption. Our model shows that this approximation is not acceptable because business-cycle volatility significantly affects the gap between average and steady-state employment and consumption. Thus, business-cycle costs are sizable: they are 50 times the amount estimated by Lucas.

Without financial frictions, workers' welfare cost falls drastically ($100 \times \tau = 0.12$, line 1, column B, Table 3).[39] The magnitude of business-cycle costs is reduced to 2.4 times Lucas' evaluation. This estimation is slightly lower than the value reported in Hairault et al. (2010) or Jung & Kuester (2011). In our model, wages are set by using a Nash bargaining solution while Hairault et al. (2010) and Jung & Kuester (2011) assume exogenous wage rigidity or sluggishness. Finally, in the model without financial frictions, under a calibration that mimics the first-order allocation (i.e., $b = 0$, no unemployment allocation and the Hosios condition $\epsilon = \psi$), welfare costs are negligible (0.02%). Welfare costs in the first-best economy are substantially lower than that which arises in presence of realistic financial and search

[39] This estimate is computed by using our model without both discounting heterogeneity and collateral constraints. We consider the parameter values in Table 1 (panels (b) and (c)). The rationale behind this approach is the following. The model with financial frictions is considered as the "true" model of the economy. By calibrating the model so as to match key financial and labor market targets, we uncover the "true" parameter values. The business-cycle costs without financial frictions are then computed with these parameter values, including the standard deviation of technological shocks ($\sigma_A = 0.0031$). If we adjust the standard deviation of technological shocks to match output volatility in the model without financial frictions, the welfare costs amount to 0.41% of permanent consumption. This remains far below the values reported in the model with financial frictions.

and matching frictions.

If we apply our computations to the data, the contrast with Lucas's results is straightforward. The level effect associated with employment only is $100 \times \frac{\bar{N}-\mathbb{E}(N)}{\bar{N}} = 3.15$, i.e. a loss of 4.7 millions jobs in 2015 and a GDP per capita loss of approximately 1520 dollars a year. Without financial frictions, the level effect entails a loss of about 100.000 jobs, and each household loses 38 dollars per year.

5.4 Sensitivity analysis

Table 4 reports quantitative results when the model is modified along one dimension at the time. Appendix D reports further details on model equations and calibration for each exercise.

Table 4: Welfare costs of business cycles: Sensitivity analysis

	Benchmark	Sluggish wage	Higher Γ	Utility function	Financial shocks	Capital as collateral	Financial shocks Capital	Capital wage and investment costs
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\tau \times 100$	2.50	-0.13	0.76	2.72	1.79	2.67	3.4	3.39

In each experiment, we adjust the volatility of aggregate shocks σ_A in order to match the output volatility from the data (1.44 as reported in Table 2).

5.4.1 Exogenous sluggish wage

Column (2) of Table 4 accounts for wage sluggishness, as initially suggested by Hall (2005*b*). The wage is determined by

$$\tilde{w}_t = \tilde{\gamma} w_t^{Nash} + (1-\tilde{\gamma})\tilde{w}_{t-1}$$

where w_t^{Nash} refers to the Nash-bargained wage from equation (12). The benchmark calibration is such that $\tilde{\gamma} = 1$, which entails wage volatility close to the data (see Table 2). We aim at illustrating[40] that wage rigidity with financial frictions can reduce welfare costs of fluctuations (see discussion in Section 4 and analytical results in Appendix C). If wage sluggishness is large enough, the model with financial frictions can actually generate welfare gains of fluctuations because firms benefit so much from expansions that average unemployment can be lower than its deterministic counterpart. Simulation results in column (2) of Table 4 illustrate this point. With a very low $\tilde{\gamma} = 0.3$, the model generates welfare *gains* of fluctuations (of 0.13%).

[40] We do not necessarily push here the idea that wage sluggishness is needed.

5.4.2 Utility function

The value of non-employment relative to the value of market work. In our model, business cycles induce a large average unemployment rate with respect to its deterministic counterpart. The extent to which this translates into greater welfare costs of business cycles depends on the value of non-employment relative to the value of work. In our utility function (equation (3)), Γ captures the additional utility derived from leisure when the worker is unemployed. As the value of being unemployed increases relative to the value of employment, high average unemployment relative to deterministic unemployment can actually please workers, in spite of lower output and lower consumption (as can be seen from the expression of τ_{Gap} in equation (24)). Indeed, with higher Γ (column (3) of Table 4), welfare costs of fluctuations goes down to 0.76% versus 2.5% in the benchmark case.

We argue that the benchmark calibration of Γ is the relevant one. It is derived at the steady state so as to be consistent with Hall & Milgrom (2008)'s unemployment benefits and steady-state labor-market characteristics (see section 5.1). In order to get a higher value of Γ, we here arbitrarily lower unemployment benefit replacement ratio to a small level of 0.5. This yields $\Gamma = 0.38$ instead of 0.18 in the benchmark calibration.

Utility function. With the benchmark utility function (3), wages are not affected by consumption-savings intertemporal choices. Hansen (1985) and Jung & Kuester (2011) consider an alternative utility function in which this is not the case. [41] Results in column (4) of Table 4 have been computed by using the following utility function [42]

$$\frac{{C_t}^{1-\sigma}}{1-\sigma} - \Gamma N_t \tag{25}$$

with $\sigma > 0$ the relative risk aversion. We compute the corresponding expression of τ. Unlike in the benchmark case, labor and consumption are separable. As a result, the expression of τ does not involve covariances between consumption and labor.[43] We also find sizeable welfare costs of fluctuations (2.72%).

[41] Differently from the benchmark model, at the steady state, the wage equation (49) in Appendix D.2 exhibits a first term, reservation wage, involving worker marginal consumption utility λ. In contrast, marginal consumption utility around the steady state in our benchmark model – captured by λ and λ^F in equation (16) – does not affect the wage.

[42] Indivisible labor creates ex-post heterogeneity across agents as some work, others don't. However, we consider a representative agent in our model. We consider only this utility function as, with indivisible labor and this utility function, the solution of the representative agent's problem can be supported by a competitive equilibrium. See Hansen (1985) and Rogerson (1988) for a formal proof.

[43] See Appendix D.2 for a description of the model, calibration and analytical computation of τ

5.4.3 Financial Shocks

In column (5) of Table 4, we report the welfare costs when the model is augmented with financial shocks. The latter are captured as shocks to parameter m in the collateral constraint (equation (9)), as in Jermann & Quadrini (2012) and Liu et al. (2013). The calibration of the shock is based on Liu et al. (2013).[44] Welfare costs in an economy with technological and financial shocks decrease to 1.79% versus 2.5% in the benchmark case. Because financial shocks introduce more uncertainty on the value that banks can recover from the collateral, the land price needs to incorporate a premium (average land price is greater than its steady-state value: $\mathbb{E}(q) > \overline{q}$). Thus, wealth increases on average, raising consumption relative to the case with technological shocks only.[45] Financial shocks improve the match between debt and land-price volatilities (see Column (2) of Table 5 in Appendix D.4), but this shock induces excess volatility in vacancies.

5.4.4 Adding capital.

Adding capital as collateral. Column (6) of Table 4 incorporates capital into the model along the lines of Liu et al. (2013).[46] The production function now includes physical capital $K_t : Y_t = A_t \left[L_t^{\phi} K_{t-1}^{1-\phi}\right]^{1-\alpha} N_t^{\alpha}$. We also consider adjustment costs on capital such that : $K_t = (1-\delta)K_{t-1} + I_t + (\Omega)/2)((I_t/I_{t-1}) - 1)^2 I_t$ where I_t denotes investment, and δ the rate of capital depreciation. We consider the same calibration of ϕ, Ω, and δ as in Liu et al. (2013). The financial constraint becomes

$$B_t + \omega V_t \leq m \left(E_t \left[q_{t+1}^k\right] K_t + E_t \left[q_{t+1}\right] L_t\right)$$

with q^k the price of capital. Notice that welfare costs here (2.67%) are greater than in the benchmark model (2.50%). Indeed, capital amplifies our basic mechanism. As employment is lower, on average, than in steady state, the marginal productivity of capital is low in the stochastic economy. The incentive to save is thus reduced. Notice also that, when capital is included into the collateral constraint, models with technological shocks only are not able to replicate the volatilities of debt and land price (see column (3) of Table 5 in Appendix D.5), thereby making the assessment of the welfare cost of fluctuations less convincing.

Finally, the introduction of both capital and financial shocks can be viewed as a solution to fit the financial indicators of the business cycle as well as those of investment (column (7)

[44]See Appendix D.3 for more details.
[45]See Appendix E.1 for a formal analysis of this mechanism using a simplified version of our model.
[46]See Appendix D.4 for more details.

of Table 4). Indeed, this proceeds in the right direction, but at the cost of excess volatility of labor market fluctuations. The reason comes from the very low sensitivity of wages. Financial frictions are too strong and thus overestimate the wage moderation induced by the credit channel.[47] Notice also that the impact of financial shocks in our model depends on whether we include capital. In economies without capital (column (5) of Table 4), financial shocks reduce welfare costs. In contrast, in presence of capital (columns (7)), financial shocks increase welfare costs. This is because financial shocks make banks' return on the collateral more risky. This uncertainty generates a premium on the borrowing constraint (i.e., an over-accumulation). This in turn generates a new motive to increase debt (B increases in the volatile economy relative to the stabilized economy), entailing a parallel increase in workers' wealth. As a result, in our benchmark model without capital consumption is on average greater than at the steady state. This is not the case when we include capital accumulation. This latter setting is characterized by decreasing returns to scale. As greater leveraging in the volatile economy induce additional capital, over-accumulation is then costly in terms of consumption.[48]

Including wage and investment costs in the financial constraint. Results in column (8) of Table 4 account for the inclusion of the wage bill and investment costs into the financial constraint, as in Millard et al. (2017) [49]. The financial constraint is now

$$B_t + \omega V_t + w_t N_t + I_t \leq m \left(E_t \left[q^k_{t+1} \right] K_t + E_t \left[q_{t+1} \right] L_t \right)$$

The welfare costs of business cycle are larger than in the benchmark case (3.39% versus 2.50%) as the financial constraint includes more elements of the firm's cost.

Finally, we report in Appendix F results on asymmetric welfare costs of business cycles: pains from recessions are larger than gains from booms. Petrosky-Nadeau & Zhang (2017) and Ferraro (2018) investigate the asymmetric behavior of the DMP model in recessions and expansions. We show that unemployment asymmetry over the business cycle is a key element also to evaluate the welfare costs of fluctuations. We compute the time-varying welfare cost and report its empirical distribution for the model with financial and search frictions and the model with search frictions only. Whatever the model, welfare costs in

[47]A solution for this shortcoming of this extension would be to consider a more complex modelling of the collateral constraint as in Liu et al. (2013), where the collateral is not the simple sum of the value of capital and land, but a weighted sum of these two assets. Given that these weights are unknown, this is left for future research.

[48]See Appendix E.2 for an analysis of this mechanism in a Mickey-Mouse model.

[49]See Appendix D.4.

recessions are larger than welfare gains in expansion. This result captures the asymmetric business cycles in our non-linear environment. In addition, the presence of financial frictions shifts the distribution of welfare costs to regions showing more asymmetry and greater losses: with FF, welfare costs are larger in recessions while welfare gains remain small in booms.

6 Conclusion

This paper provides a quantitative assessment of welfare costs of fluctuations in a labor market search model with financial frictions *à la* Kiyotaki & Moore (1997). Because of labor market search frictions, fluctuations generate a higher average unemployment rate relative to its steady-state value, thereby increasing the welfare cost of fluctuations. Financial frictions amplify this mechanism, together with the associated welfare costs. We show that business-cycle costs are sizable: they are 50 times the amount estimated by Lucas. Without financial constraints, the magnitude of business-cycle costs is reduced to 2.4 times Lucas's evaluation.

Our model is able to replicate business-cycle volatilities on both labor and financial markets. In particular, it reproduces the high degree of responsiveness of the job-finding rate throughout the business cycle. Indeed, financial frictions entail wage sluggishness that helps the model match the large changes in job-finding rates observed in the data; at the same time, it allows for real wage volatility as observed in the data.

References

Adjemian, S., Bastani, H., Karame, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. & Villemot, S. (2014), Dynare, Reference manual, Cepremap.

Adjemian, S., Karame, F. & Langot, F. (2016), On non-linearities in the unemployment, Mimeo, Cepremap.

Aiyagari, S. (1994), 'Uninsured idiosyncratic risk and aggrigate saving', *Quarterly Journal of Economics* pp. 659–684.

Andolfatto, D. (1996), 'Business cycles and labor-market search', *The American Economic Review* (1), 112–132.

Barro, R. (2006), 'Rare disasters and asset markets in the twentieth century', *Quarterly Journal of Economics* **121**, 823–866.

Barron, J., Berger, M. & Black, D. (1997), 'Employer search, training, and vacancy duration', *Economic Inquiry* **35**, 167–192.

Barron, J. & Bishop, J. (1985), 'Extensive search, intensive search, and hiring costs: New evidence on employer hiring activity', *Economic Inquiry* **23**(3), 363–382.

Beaudry, P. & Pages, C. (2001), 'The cost of business cycles and the stabilization value of unemployment insurance', *European Economic Review* **45**, 1545–1572.

Bernanke, B., Gertler, M. & Gilchrist, S. (1999), The financial accelerator in a quantitative business cycle framework, *in* J. B. Taylor & M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1, Elservier, pp. 1341–1393.

Blanchard, O. & Gali, J. (2010), 'Labor markets and monetary policy: A new keynesian model with unemployment', *American Economic Journal: Macroeconomics* **2**(2), 1–30.

Chéron, A. & Langot, F. (2004), 'Labor market search and real business cycles: reconciling nash bargaining with the real wage dynamics', *Review of Economic Dynamics* pp. 476–493.

Christiano, L., Motto, R. & Rostagno, M. (2010), Financial factors in economic fluctuations, ECB working paper 1192, ECB.

Chugh, S. (2013), 'Costly external finance and labor market dynamics', *Journal of Economic Dynamics and Control* **37**(12), 2882–2912.

Diamond, P. (1982), 'Wage determination and efficiency in search equilibrium', *Review of Economic Studies* **49**, 217–227.

Dromel, N., Kolakez, E. & Lehman, E. (2010), 'Credit constraints and the persistence of unemployment', *Labour Economics* **17**(5), 823–834.

Ferraro, D. (2018), 'The asymmetric cyclical behavior of the u.s. labor market', *Review of Economic Dynamics* **30**, 145–162.

Garin, J. (2015), 'Borrowing constraints,collateral fluctuations, and the labor market', *Journal of Economic Dynamics and Control* **57**, 112–130.

Gertler, M. & Kiyotaki, N. (2010), Financial intermediation and credit policy in business cycle analysis, *in* B. Friedman & M. Woodford, eds, 'Handbook of Monetary Economics', Vol. 3, Elsevier, pp. 547–599.

Greenwood, J., Hercowitz, Z. & Huffman, G. (1988), 'Investment, capacity utilization, and the real business cycle', *The American Economic Review* **78**(3), 402–417.

Hagendorn, M. & Manovskii, I. (2008), 'The cyclical behavior of equilibrium unemployment and vacancies revisited', *American Economic Review* **98**(4), 1692–1706.

Hairault, J.-O., Langot, F. & Osotimehin, S. (2010), 'Matching frictions, unemployment dynamics and the cost of business cycles', *Review of Economic Dynamics* **13**(4), 759–779.

Hall, R. (2005*a*), 'Employment efficiency and sticky wages: Evidence from flows in the labor market', *Review of Economics and Statistics* **87**(3), 397–407.

Hall, R. (2005*b*), 'Employment fluctuations with equilibrium wage stickiness', *American Economic Review* **95**, 50–65.

Hall, R. (2017), 'High discounts and high unemployment', *American Economic Review* **107**(2), 305–330.

Hall, R. & Milgrom, P. (2008), 'The limited influence of unemployment on the wage bargain', *American Economic Review* **98**(4), 1653–1674.

Hansen, G. (1985), 'Indivisible labor and the business cycle', *Journal of Monetary Economics* **16**(3), 309–327.

Iacoviello, M. (2005), 'Housing prices, borrowing constraints and monetary policy in the business cycle', *American Economic Review* **95**(3), 739–764.

Jermann, U. & Quadrini, V. (2012), 'Macroeconomic effects of financial shocks', *American Economic Review* **102**(1), 238–271.

Jung, P. & Kuester, K. (2011), 'The (un)importance of unemployment fluctuations for the welfare cost of business cycles', *Journal of Economic Dynamics and Control* **35**(10), 1744–1768.

Kiyotaki, N. & Moore, J. (1997), 'Credit cycle', *Journal of Political Economy* **105**(2), 211–248.

Krause, M. & Lubik, T. A. (2007), 'The (ir)relevance of real wage rigidity in the new keynesian model with search frictions', *Journal of Monetary Economics* **54**(4), 706–727.

Krebs, . (2003), 'Growth and welfare effects of business cycles in economies with idiosyncratic human capital risk.', *Review of Economic Dynamics* **6**, 846–868.

Krusell, P., Mukoyama, T. & Sahin, A. (2010), 'Labour-market matching with precautionary savings and aggregate fluctuations', *Review of Economic Studies* **77**(4), 1477–1507.

Liu, Z., Miao, J. & Zha, T. (2016), 'Land prices and unemployment', *Journal of Monetary Economics* **80**, 86–105.

Liu, Z., Wang, P. & Zha, T. (2013), 'Land-price dynamics and macroeconomic fluctuations', *Econometrica* **81**(3), 1147–1184.

Lucas, R. (1987), *Models of Business Cycles*, Blackwell Publishing.

Lucas, R. (2003), 'Macroeconomic priorities', *American Economic Review* **93**(1), 1–14.

MacKay, A. & Reis, R. (2008), 'The brevity and violence of contractions and expansions', *Journal of Monetary Economics* **55**, 738–751.

Millard, S., Varadi, A. & Yashiv, E. (2017), Interactions of financial and real frictions along the business cycle, Mimeo, Bank of England.

Monacelli, T., Quadrini, V. & Trigari, A. (2011), Financial markets and unemployment, NBER Working Paper Series 17389, NBER.

Mortensen, D. (1982), The matching process as a noncooperative bargaining game, *in* J. J. McCall, ed., 'The Economics of Information and Uncertainty', University of Chicago Press.

Petrongolo, B. & Pissarides, C. (2001), 'Looking into the black box : A survey of the matching function', *Journal of Economic Literature* **39**, 390–431.

Petrosky-Nadeau, N. (2013), 'Credit, vacancies and unemployment fluctuations', *Review of Economic Dynamics* **17**(2), 191–205.

Petrosky-Nadeau, N. & Zhang, L. (2017), 'Solving the dmp model accurately', *Quantitative Economics* **8**, 611–650.

Petrosky-Nadeau, N., Zhang, L. & Kuehn, L. (2018), 'Endogenous disasters', *American Economic Review* **108**(8), 2212–2245.

Pissarides, C. (1985), 'Short-run dynamics of unemployment, vacancies, and real wages', *American Economic Review* **75**, 676–690.

Pissarides, C. (2009), 'The unemployment volatility puzzle: Is wage stickiness the answer?', *Econometrica* **77**(5), 1339–1369.

Pissarides, C. (2011), 'Equilibrium in the labor market with search frictions', *American Economic Review* **101**(4), 1092–1105.

Quadrini, V. (2011), 'Financial frictions in macroeconomic fluctuations', *Economic Quarterly* **97**(3), 209–254.

Rogerson, R. (1988), 'Indivisible labor, lotteries and equilibrium', *Journal of Monetary Economics* **21**, 3–16.

Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review* **95**(1), 25–49.

Shimer, R. (2012), 'Reassessing the ins and outs of unemployment', *Review of Economic Dynamics* **15**, 127–148.

Storesletten, K., Telmer, C. & Yaron, A. (2001), 'The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk', *European Economic Review* **45**, 1311–1339.

Wasmer, E. & Weil, P. (2004), 'The macroeconomics of labor and credit market imperfections', *American Economic Review* **94**(4), 944–963.

Yashiv, E. (2007), 'Labor search and matching in macroeconomics', *European Economic Review* **51**, 1859–1895.

Appendix For Online Publication

A Data

Aggregate data. The following quarterly time series come FRED database, the Federal Reserve Bank of Saint Louis' website (1976Q1-2013Q1). y is Real Gross Domestic Product from the FRED database (GDPC96) divided by the Civilian Non institutional Population from the FRED database (CNP16OV). c is Real Personal Consumption Expenditures from the FRED data-base (PCECC96) divided by the Civilian Non institutional Population from the FRED database (CNP16OV)

Labor market data. w is Compensation of Employees: Wages & Salary Accruals from the FRED database (WASCUR) divided by Civilian Employment (CE16OV). N is Civilian Employment (CE16OV) divided by Civilian Non institutional Population. U is FRED, Civilian Unemployment Rate (UNRATE), Percent, quarterly, Seasonally Adjusted. The latter time series are taken from the FRED database. As for the time series of the job finding rate, we use monthly CPS data from January 1976 to March 2013. We follow all the steps described in Shimer (2012). As in Shimer (2012), we correct for time aggregation and take quarterly averages of monthly observations. V are vacancies Total Nonfarm, Total US Job Openings (JTS00000000JOL), Seasonally Adjusted Monthly data from BLS. We take quarterly averages of this time series that is available only from December 2000 onwards.

Debt, loan Rate and land price. We follow Jermann & Quadrini (2012). Financial data come from the Flow of Funds Accounts of the Federal Reserve Board. The debt stock is constructed by using the cumulative sum of net new borrowing measured by the 'Net increase in credit markets instruments of non financial business'[50]. Since the constructed stock of debt is measured in nominal terms, it is deflated by the price index for business value added from NIPA. The initial (nominal) stock of debt is set to 94.12, which is the value reported in the balance sheet data from the Flow of Funds in 1952.I for the nonfarm non financial business. The cumulative sum starts in 1952, which, as in Jermann & Quadrini (2012), is not likely to affect our data starting on January 1976. R is the log of 1+ the Bank Prime Loan Rate (MPRIME) (used as a reference for short-term business loan) from

[50]Nonfinancial business; credit market instruments; liability; Net increase in credit markets instruments of non financial business, millions of dollars (nominal). FA144104005.Q, F.101 Line 28.

the FRED database. Finally, we use as a proxy for q the price index for residential land as computed by Liu et al. (2013).

Cyclical components of the data: All data are quarterly (from 1976:Q1 through 2013:Q1), in logs, $HP(\lambda = 1600)$ filtered and multiplied by 100 in order to express them in percent deviation from steady state. Ψ is the job finding rate computed from Monthly CPS data from January 1976 to March 2013 using Shimer (2012)'s methodology. It measures the probability for an unemployed worker to find a job. As for financial data on debt and interest rate, we follow Jermann & Quadrini (2012). We finally check that our financial and labor market time series are consistent with the data available on line for Shimer (2012) and Jermann & Quadrini (2012).

B Model

B.1 Household

Each household knows that the evolution of S follows (2), so that (5) can be written as:

$$N_t = (1-s)N_{t-1} + \Psi_t \left(1 - (1-s)N_{t-1}\right) \tag{26}$$

The dynamic problem of a typical household can be written as follows

$$\mathcal{W}(\Omega_t^H) = \max_{C_t^n, C_t^u, B_t} \left\{ N_t U(C_t^n) + (1-N_t)U(C_t^u + \Gamma) + \mu \mathbb{E}_t \mathcal{W}(\Omega_{t+1}^H) \right\}$$

subject to (26) and (4), given the initial conditions on state variables (N_0, B_0) and $\Omega_t^H = \{N_{t-1}, \Psi_t, w_t, b_t, T_t, B_{t-1}\}$, the vector of variables taken as given by households. Let λ_t be the shadow price of the budget constraint. The first order conditions associated with consumption choices are

$$(C_t^n)^{-\sigma} = (C_t^u + \Gamma)^{-\sigma} = \lambda_t \tag{27}$$

Hence $U_t(C_t^n) = U_t(C_t^u + \Gamma)$. The first order condition associated to bond holdings reads:

$$-\lambda_t + \mu \mathbb{E}_t \left[R_t \lambda_{t+1}\right] = 0 \tag{28}$$

B.2 Entrepreneur

The firm's program is

$$\mathcal{W}(\Omega_t^F) = \max_{C_t^F, L_t, B_t, V_t, N_t} \left\{U\left(C_t^F\right) + \beta \mathbb{E}_t\left[\mathcal{W}(\Omega_{t+1}^F)\right]\right\} \tag{29}$$

$$s.t. \quad \begin{cases} -C_t^F - R_{t-1}B_{t-1} - q_t\left[L_t - L_{t-1}\right] - w_t N_t - \bar{\omega} V_t & \\ \qquad +Y_t\left(A_t, L_{t-1}, N_t\right) + B_t = 0 & (\lambda_t^F) \\ -B_t - \bar{\omega} V_t + m\mathbb{E}_t\left[q_{t+1}L_t\right] = 0 & (\lambda_t^F \varphi_t) \\ -N_t + (1-s)N_{t-1} + \Phi_t V_t = 0 & (\xi_t) \end{cases}$$

given the initial conditions N_0, B_0, where $\Omega_t^F = \{N_{t-1}, \Psi_t, w_t, b_t, \pi_t, T_t, B_{t-1}, L_{t-1}\}$ is the vector of variables taken as given by firms. Letting λ_t^F, $\lambda_t^F \varphi_t$, and ξ_t be the Lagrange multipliers associated to (7), (9) and (10) the first order conditions of the firm problem read:

$$U'\left(C_t^F\right) = \lambda_t^F \tag{30}$$

$$\lambda_t^F q_t = \beta \mathbb{E}_t\left[\lambda_{t+1}^F\left(q_{t+1} + \frac{\partial Y_{t+1}}{\partial L_t}\right)\right] + \lambda_t^F \varphi_t m \mathbb{E}_t\left[q_{t+1}\right] \tag{31}$$

$$(1-\varphi_t)\lambda_t^F = \beta \mathbb{E}_t \lambda_{t+1}^F R_t \tag{32}$$

$$\xi_t = \lambda_t^F \bar{\omega} \frac{(1+\varphi_t)}{\Phi_t} \tag{33}$$

$$\xi_t = \lambda_t^F\left[\left(\frac{\partial Y_t}{\partial N_t}\right) - w_t\right] + (1-s)\beta \mathbb{E}_t\left[\xi_{t+1}\right] \tag{34}$$

where (30) is the condition associated to consumption, (33) the one on vacancy posting, and (34) on employment[51]. Equation (31) is the one associated with land accumulation. It implies that, in equilibrium, the value of current marginal utility of consumption needs to equal the indirect value of utility deriving from land accumulation, i.e.: *i)* the value of future consumption utility deriving from reselling land in the next period, $\beta \mathbb{E}_t \lambda_{t+1}^F q_{t+1}$; *ii)* the future consumption utility arising from the product of land, $\beta \mathbb{E}_t \lambda_{t+1}^F \frac{\partial Y_{t+1}}{\partial L_t}$; *iii)* the additional utility arising from current consumption related to the effect of land in loosening the collateral constraint, $\varphi_t m \lambda_t^F \mathbb{E}_t\left[q_{t+1}\right]$.

Equation (32) is a modified Euler equation. When the collateral constraint is not binding, φ_t is equal to zero and we recover the standard Euler equation. When the debt limit is binding, $\varphi_t > 0$ and $\varphi_t = 1 - \beta \frac{\mathbb{E}_t \lambda_{t+1}^F R_t}{\lambda_t^F}$ implying that firms' marginal utility of current consumption is greater than their discounted marginal utility of future consumption. Impatient firms choose thus to increase consumption up to the limit imposed by (9).

[51] Note that, entrepreneurs are not risk neutral. With $\lambda_t^F = 1$, we recover the canonical search model.

Using equation (33) into equation (34) yields the job creation curve (equation (11) in section 3.3).

B.3 The wage curve

From the household's intertemporal program, one gets:

$$\begin{aligned}\mathcal{V}_t^H &= \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}} = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_t}\frac{\partial N_t}{\partial N_{t-1}} + \mu \mathbb{E}_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\frac{\partial N_t}{\partial N_{t-1}} \\ &= \left[U_t(C_t^n) - U_t(C_t^u + \Gamma) + \lambda_t w_t - \lambda_t b_t - \lambda_t\left(C_t^n - C_t^u\right)\right]\frac{\partial N_t}{\partial N_{t-1}} + \mu \mathbb{E}_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\frac{\partial N_t}{\partial N_{t-1}}\end{aligned}$$

With $U_t(C_t^n) = U_t(C_t^u + \Gamma)$, we have

$$\mathcal{V}_t^H = \left[\lambda_t w_t - \lambda_t(b_t + \Gamma)\right]\frac{\partial N_t}{\partial N_{t-1}} + \mu \mathbb{E}_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\frac{\partial N_t}{\partial N_{t-1}}$$

Where, from (26) $\frac{\partial N_t}{\partial N_{t-1}} = (1-s)\left(1-\Psi_t\right)$, so that

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)\left(1-\Psi_t\right)\left[w_t - (b_t + \Gamma) + \mu \mathbb{E}_t \left(\frac{1}{\lambda_t}\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\right] \tag{35}$$

From the firms' program $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}} = \xi_t\left(1-s\right)$ where $\xi_t = \lambda_t^F \bar{\omega}\frac{(1+\varphi_t)}{\Phi_t}$, thus:

$$\begin{aligned}\frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} &= (1-s)\frac{\bar{\omega}}{\Phi_{t+1}}\lambda_{t+1}^F\left(1+\varphi_{t+1}\right) \\ \frac{\mathcal{V}_t^F}{\lambda_t^F} &= (1-s)\frac{\bar{\omega}}{\Phi_t}\left(1+\varphi_t\right)\end{aligned}$$

Then, using (11) we obtain:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s)\left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta \mathbb{E}_t \left(\frac{1}{\lambda_t^F}\frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t}\right)\right]$$

Therefore, the surpluses are, respectively:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s)\left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^F}{\lambda_t^F}\frac{\mathcal{V}_{t+1}^F}{\lambda_{t+1}^F}\right)\right] \tag{36}$$

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)\left(1-\Psi_t\right)\left[w_t - (b_t + \Gamma) + \mu \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_{t+1}}\frac{\mathcal{V}_{t+1}^H}{\lambda_t}\right)\right] \tag{37}$$

By maximizing the Nash product with respect to the wage, we obtain $\left(\frac{\mathcal{V}_t^H}{\lambda_t}\right) = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right) \frac{(1-\epsilon)(1-\Psi_t)}{\epsilon}$. By substituting for (36) and (37), and rewriting it, we obtain the wage curve (equation (12) in section 3.4).

C Analysis of the simplified model

This Appendix analyzes the simplified model in section 4.

C.1 The responsiveness of labor market tightness to the business cycle (θ')

We examine here proposition 1. In order to derive θ', we need to solve the labor market equilibrium (ie. find θ), as the intersection of the job creation and the wage curve. We will use the simplified version of the model described in section 4.

Let us denote $\Lambda = \varrho\frac{\chi}{\overline{\omega}}$ with $\varrho = 1/(1-\beta(1-s))$. The job creation condition leads to

$$\begin{array}{lrcl} (noFF) & \theta(y)^{1-\psi} & = & \Lambda(y - w(\theta(y), y)) \\ (FF) & \theta(y)^{1-\psi} & = & \Lambda\dfrac{y - w(\theta(y), \varphi(y), y)}{1+\varphi(y)} \end{array}$$

where "noFF" refer to the standard DMP model and "FF" the DMP model with financial frictions. In booms, access to credit is cheaper (φ falls), which increases the firm's gain from a filled vacancy. The wage setting rules are

$$\begin{array}{lrcl} (noFF) & w(\theta(y), y) & = & \epsilon(b+\Gamma) + (1-\epsilon)\left(y + \Sigma(\theta(y), y)\right) \\ (FF) & w(\theta(y), \varphi(y), y) & = & \epsilon(b+\Gamma) + (1-\epsilon)\left(y + \Sigma(\theta(y), \varphi(y), y)\right) \end{array}$$

where outside search options are given by

$$\begin{array}{lrcl} (noFF) & \Sigma(\theta(y), y) & = & (1-s)\beta\overline{\omega}\theta(y) \\ (FF) & \Sigma(\theta(y), \varphi(y), y) & = & (1-s)\dfrac{\beta}{1-\varphi(y)}\left(-\dfrac{\overline{\omega}(1+\varphi(y))}{\Phi(\theta(y))}\varphi(y) + \overline{\omega}(1+\varphi(y))\theta(y)\right) \end{array}$$

Given that what matters for the firm at the time of hiring is the discounted sum of labor costs, ie. $w/(1+\varphi(y))$, labor market tightness is then a function of the discounted sum of outside search options, i.e.

$$\frac{\Sigma(\theta(y),\varphi(y),y)}{1+\varphi(y)} = (1-s)\frac{\beta}{1-\varphi(y)}\left(-\frac{\overline{\omega}}{\Phi(\theta(y))}\varphi(y)+\overline{\omega}\theta(y)\right),$$

Interestingly, with financial frictions there are two opposing forces incorporated into $\Sigma/(1+\varphi)$: on the one hand, in booms, labor market tightness increases, which tends to raise the opportunity cost of search activities (last term in $\Sigma/(1+\varphi)$ is common to all DMP models); on the other hand, congestion externalities make recruitment longer in booms ($1/\Phi$ increases), which lengthens the duration of the credit as hiring costs are financed through loans. Because of impatience gaps, heterogenous valuations of future gains from the match lead impatient entrepreneurs to be reluctant to accept higher wages (first term in $\Sigma/(1+\varphi)$).

Finally, we consider the ad hoc reduced form for financial frictions, $\varphi(y) = 1 - \frac{\beta}{\mu}y^{\zeta}$. After replacing the wage into the job creation conditions, we obtain

$$\begin{array}{lrcl} (noFF) & \underbrace{\theta(y)^{1-\psi}+\Omega\theta(y)}_{H(\theta(y))} & = & \Upsilon(y-(b+\Gamma)) \\ (FF) & \underbrace{\theta(y)^{1-\psi}+\Omega\theta(y)}_{H(\theta(y))} & = & \Upsilon(y-(b+\Gamma))\Delta(y) - \Omega\underbrace{\frac{\varphi(y)}{1-\varphi(y)}\theta(y)\left(1-\frac{1}{\Psi(\theta(y))}\right)}_{=G(y,\theta(y))} \end{array}$$

with $\Upsilon = \Lambda\epsilon$, $\Omega = \Lambda(1-\epsilon)(1-s)\beta\overline{\omega}$ and $\Delta(y) = \frac{1}{2-\frac{\beta}{\mu}y^{\zeta}}$.

The function $\Delta(y)$ can be interpreted as the credit multiplier for hiring decisions. It satisfies $\Delta' = \frac{\zeta\frac{\beta}{\mu}y^{\zeta-1}}{\left(2-\frac{\beta}{\mu}y^{\zeta}\right)^2}$ and $\Delta'' = \zeta\frac{\beta}{\mu}y^{\zeta-1}\left(2-\frac{\beta}{\mu}y^{\zeta}\right)\frac{(\zeta-1)\left(2-\frac{\beta}{\mu}y^{\zeta}\right)+2\zeta y^{\zeta-1}}{\left(2-\frac{\beta}{\mu}y^{\zeta}\right)^4}$, that implies, for $y=1$ and $\beta \approx \mu$ (restrictions satisfied by our calibration), $\Delta' \to \zeta$ and $\Delta'' \to \zeta(3\zeta-1)$. Notice that if $\zeta > 1/3$[52], then $\Delta'' > 0$, i.e. the credit multiplier is convex.

Finally, the function $G(y,\theta(y))$ measures the impact of labor market tightness on the credit costs included in the wage equation. Given that $\Psi(\theta(y))$ is the job finding rate, close to 0.4 (see Table 1), the term $\left(1-\frac{1}{\Psi(\theta(y))}\right)$ is negative, so that

- in boom, $G(y,\theta) > 0$ (as $y > 1$, then $\frac{\varphi(y)}{1-\varphi(y)} < 0$)
- in recession, $G(y,\theta) < 0$ (as $y < 1$, then $\frac{\varphi(y)}{1-\varphi(y)} > 0$)

Given the definition of G, we have $G'_y(y,\theta) = -\zeta\frac{\mu}{\beta}y^{-\zeta-1}\left(\theta(y)-\frac{1}{\chi}\theta(y)^{1-\psi}\right)$ and $G'_\theta(y,\theta) = \left(\frac{\mu}{\beta}y^{-\zeta}-1\right)\left(1-\frac{1-\psi}{\chi}\theta(y)^{-\psi}\right)$. For $y=1$ and $\beta \approx \mu$, we deduce that $G'_y(y,\theta) \to -\zeta\theta\left(1-\frac{1}{\Psi}\right) >$

[52] This is a relatively light restrictions and it is verified by our full fledged model

0 and $G'_\theta(y,\theta) \to 0$.

The responses of labor market tightness θ to aggregate productivity y are given by

$$\begin{aligned}
\theta'|_{noFF} &= \frac{\Upsilon}{(1-\psi)\theta(y)^{-\psi}+\Omega} = \frac{\Upsilon}{H'(\theta)} \\
\theta'|_{FF} &= \frac{\Upsilon[\Delta(y)+(y-(b+\Gamma))\Delta'(y)]-\Omega G'_y(y,\theta)}{H'(\theta)+\Omega G'_\theta(y,\theta)} \underset{\substack{y=1,\\ \beta\approx\mu}}{\longrightarrow} \frac{\Upsilon[1+(y-(b+\Gamma))\zeta]+\Omega\zeta\theta\left(1-\frac{1}{\Psi}\right)}{H'(\theta)}
\end{aligned}$$

If $\Upsilon(y-(b+\Gamma))\zeta+\Omega\zeta\theta\left(1-\frac{1}{\Psi}\right)>0$ then $\theta'|_{FF}>\theta'|_{noFF}>0$.

Notice that this parameter restriction relates the impact of an additional borrowed dollar (to finance hiring activities) to hiring incentives and their marginal cost. The parameter restriction can be rewritten as $\Upsilon y > \Upsilon(b+\Gamma)+\Omega\theta\left(\frac{1}{\Psi}-1\right)$. The term on the LHS captures the incentives to create jobs, i.e., the discounted sum of marginal productivity. The RHS summarizes the marginal costs of this hiring: i) those associated with the reservation wage $(b+\Gamma)$ and ii) those generated by search externalities on credit costs ($\theta\left(\frac{1}{\Psi}-1\right)$, which are incorporatd in the wage bargaining). The parameter restriction suggests that the increase of search externalities on credit costs must be lower than the increase of the discounted sum of productivity net of the reservation wage, $\Upsilon(y-(b+\Gamma))$. Indeed, during booms the wage increase must be limited such that hiring incentives remain large.

After simplifying this condition, we obtain $\left[1-\frac{1}{\varrho(1-\epsilon)(1-s)\beta}\right]<2\Psi$, which is always satisfied by our range of parameters.

The case with rigid wages. In this particular case of the model, $w=\overline{w}$. Labor market tightness is determined only by hiring decisions given by the job creation conditions

$$\begin{aligned}
(noFF) \qquad \theta(y)^{1-\psi} &= \Lambda(y-\overline{w}) \\
(FF) \qquad \theta(y)^{1-\psi} &= \Lambda(y-\overline{w})\Delta(y)
\end{aligned}$$

implying, for $y=1$ and $\beta\approx\mu$,

$$\begin{aligned}
\theta'|_{noFF} &= \frac{\Lambda}{(1-\psi)\theta(y)^{-\psi}} = \frac{\Lambda}{1-\psi}\theta(y)^{\psi} \\
\theta'|_{FF} &= \Lambda\frac{1+(y-\overline{w})\zeta}{(1-\psi)\theta(y)^{-\psi}} = \frac{\Lambda}{1-\psi}\theta(y)^{\psi}[1+\zeta(y-\overline{w})]
\end{aligned}$$

We deduce that the result still holds: $\theta'|_{FF}>\theta'|_{noFF}>0$ because $\zeta(y-\overline{w})>0$.

C.2 The convexity vs. concavity of the labor market tightness (θ'')

We examine here proposition 2. Given the expressions of $\theta'|_{FF}$ and $\theta'|_{noFF}$, we deduce the curvature of the hiring decision rules. Hence, we have

$$\begin{aligned}
\theta''|_{noFF} &= -\frac{\Upsilon^2 H''(\theta)}{H'(\theta)^3} > 0 \\
\theta''|_{FF} &= \frac{\left\{ \begin{array}{l} \left[\Upsilon(2\Delta' + (y-(b+\Gamma))\Delta'') - \Omega\left(G'_{yy} + G'_{y\theta}\theta'\right)\right](H' + \Omega G'_\theta) \\ -\left(\Upsilon[\Delta + (y-(b+\Gamma))\Delta'] - \Omega G'_y\right)\left[H''\theta' + \Omega\left(G'_{\theta y} + G'_{\theta\theta}\theta'\right)\right] \end{array} \right\}}{\left(H' + \Omega G'_\theta\right)^2}
\end{aligned}$$

Recall that $H(\theta(y)) = \theta(y)^{1-\psi} + \Omega\theta(y)$ so that $H'(\theta) > 0$ and $H''(\theta) < 0$. As a result, $\theta''|_{noFF}$ is always positive. The standard DMP model only exhibits θ-convexity.

With financial frictions, the sign of $\theta''|_{FF}$ is ambiguous. Given the definition of G, we have

$$\begin{aligned}
G''_{yy} &= \zeta(\zeta+1)\tfrac{\mu}{\beta}y^{-\zeta-2}\left(\theta(y) - \tfrac{1}{\chi}\theta(y)^{1-\psi}\right) && \to \zeta(\zeta+1)\theta\left(1 - \tfrac{1}{\Psi}\right) < 0 \\
G''_{y\theta} &= -\zeta\tfrac{\mu}{\beta}y^{-\zeta-1}\left(1 - \tfrac{1-\psi}{\chi}\theta(y)^{-\psi}\right) && \to -\zeta\left(1 - \tfrac{1-\psi}{\Psi}\right) > 0 \\
G''_{\theta y} &= -\zeta\tfrac{\mu}{\beta}y^{-\zeta-1}\left(1 - \tfrac{1-\psi}{\chi}\theta(y)^{-\psi}\right) && \to -\zeta\left(1 - \tfrac{1-\psi}{\Psi}\right) > 0 \\
G''_{\theta\theta} &= \left(\tfrac{\mu}{\beta}y^{-\zeta} - 1\right)\psi\tfrac{1-\psi}{\chi}\theta(y)^{-\psi-1} && \to 0
\end{aligned}$$

Thus, we deduce that $\theta''|_{FF}$ can be approximated by

$$H'\theta'' = \underbrace{\Upsilon(2\zeta + (y-(b+\Gamma))\zeta(3\zeta-1)) - \Omega G'_{yy}}_{(+)} - \theta'\underbrace{\left(2\Omega G'_{y\theta} + H''\theta'\right)}_{(+/-)}$$

If $2\Omega G'_{y\theta} + H''\theta' > 0$, then we can have $\theta'' < 0$. The threshold value $\widetilde{\zeta}$ which guarantees that this restriction holds is

$$\zeta > \underbrace{\frac{\Upsilon}{\Omega\left(\frac{\Psi}{\chi}\right)^{\frac{1}{\psi}}\left[\left(1 + \frac{\Omega}{(1-\psi)\chi}\Psi\right)\frac{2}{\psi}\left(\frac{1-\psi}{\Psi} - 1\right) + \left(\frac{1}{\Psi} - 1\right)\right] - \Upsilon(y-(b+\Gamma))}}_{=\widetilde{\zeta}}$$

If $\zeta > \widetilde{\zeta}$, ie. when FF are large enough (credit cost φ is responsive enough to aggregate shocks), then it is possible to have a θ-concavity.

The case with rigid wages. The endogenous mechanism entailing wage rigidity is the main channel allowing us to have a θ-concavity in the model with FF. Therefore, introducing rigid wages into the model with FF can eliminate it. We explore here this point. The expressions of $\theta'|_{FF}$ and $\theta'|_{noFF}$ in the case of rigid real wages read as:

$$\begin{aligned}
\theta''|_{noFF} &= \psi\left[\frac{\Lambda}{1-\psi}\right]^2\theta^{2\psi-1} \\
\theta''|_{FF} &= \psi\left[\frac{\Lambda}{1-\psi}\right]^2\theta^{2\psi-1}(\Delta+(y-\overline{w})\Delta')^2+\frac{\Lambda}{1-\psi}[2\Delta'+(y-\overline{w})\Delta''](1-\psi)\theta^{\psi}
\end{aligned}$$

Given that $(\Delta+(y-\overline{w})\Delta')\rightarrow(1+(y-\overline{w})\zeta)>1$ when $y\rightarrow 1$ and $\beta\approx\mu$, and $(y-\overline{w})\Delta''](1-\psi)\theta^{\psi}>0$, we deduce that $0<\theta''|_{noFF}<\theta''|_{FF}$. This confirms our intuition. FF with rigid wages are not consistent with θ-concavity. Interestingly, the model is characterized by more θ-convexity than a standard DMP model. The large convexity of $\theta''|_{FF}$ suggests that rigid real wages with FF can lead to welfare gains of cycles because they push up the average labor market tightness above its steady-state value.

D Sensitivity analysis

D.1 Wage sluggishness

The wage sluggishness is such that

$$\tilde{w}_t=\tilde{\gamma}w_t^{Nash}+(1-\tilde{\gamma})\tilde{w}_{t-1}$$

This wage is the relevant cost in the firm's decisions. The steady state is not modified by this new equation. The model is then simulated using the benchmark calibration.

D.2 Utility function

D.2.1 Greater Γ

We simulate the model with a greater value of Γ in our utility function (3). The value of Γ is derived at the steady state by using the Nash bargained wage equation given i) the calibrated level of unemployment benefits and ii) the bargaining power and wage (determined at the steady state by the job creation curve). In order to increase Γ, we lower the level of unemployment benefits from 0.7 to 0.5. The resulting value of Γ increases from 0.1848 in the benchmark calibration to 0.3848. The volatility of technological shocks is adjusted to match the volatility of output.

D.2.2 Alternative utility function

Wage bargaining now depends on intertemporal consumption-savings choices We now demonstrate that with the alternative formulation of utility wage bargaining depends on intertemporal consumption-savings choices. The wage determination is aimed at maximizing the total surplus from the match, i.e.:

$$\max_{w_t} \mathcal{S}_t = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right)^{\epsilon} \left(\frac{\mathcal{V}_t^H}{\lambda_t}\right)^{1-\epsilon}$$

with $\mathcal{S}_t$ the total surplus of a match, $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}}$ the marginal value of a match for a firm and $\mathcal{V}_t^H = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}}$ the marginal household's surplus from an established employment relationship. ϵ denotes the firm's share of a job's value, i.e., firms' bargaining power.

Household

$$\mathcal{W}(\Omega_t^H) = \max_{C_t} \left\{ U(C_t) - N_t\Gamma + \mu E_t \mathcal{W}(\Omega_{t+1}^H) \right\}$$

subject to

$$C_t + B_t \leq R_{t-1}B_{t-1} + N_t w_t + (1 - N_t)b_t + T_t \tag{38}$$

From the household's intertemporal program, one gets:

$$\mathcal{V}_t^H \quad = \quad [-\Gamma + \lambda_t w_t - \lambda_t b_t] \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}}$$

Households labor supply evolves as follows:

$$N_t = (1 - s)N_{t-1} + \Psi_t S_t \tag{39}$$

Each household knows that the evolution of S follows (2), so that (5) can be written as:

$$N_t = (1 - s)N_{t-1} + \Psi_t (1 - (1 - s)N_{t-1}) \tag{40}$$

Where, from (40):

$$\frac{\partial N_t}{\partial N_{t-1}} = (1-s)(1-\Psi_t)$$

so that

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)(1-\Psi_t)\left[-\frac{\Gamma}{\lambda_t} + (w_t - b_t) + \mu E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{\mathcal{V}_{t+1}^H}{\lambda_{t+1}}\right)\right] \tag{41}$$

hence

$$\frac{\partial\left(\frac{\mathcal{V}_t^H}{\lambda_t}\right)}{\partial w_t} = (1-s)(1-\Psi_t) \tag{42}$$

Firm

$$\begin{array}{rcl} \mathcal{W}(\Omega_t^F) & = & \max\limits_{C_t^F, V_t, N_t, B_t} \left\{U\left(C_t^F\right) + \beta E_t\left[\mathcal{W}(\Omega_{t+1}^F)\right]\right\} \\ s.t. & & \left\{\begin{array}{rcll} -C_t^F - R_{t-1}B_{t-1} - q_t\left[L_t - L_{t-1}\right] - w_t N_t - \bar{\omega}V_t & & & \\ +Y_t\left(A_t, L_{t-1}, N_t\right) + B_t & = & 0 & (\lambda_t^F) \\ -B_t - \xi_V \bar{\omega} V_t + mE_t\left[q_{t+1}L_t\right] & = & 0 & (\lambda_t^F \varphi_t) \\ -N_t + (1-s)N_{t-1} + \Phi_t V_t & = & 0 & (\xi_t) \end{array}\right. \end{array} \tag{43}$$

given the initial conditions N_0, B_0, where $\Omega_t^F = \{N_{t-1}, \Psi_t, w_t, h_t, b_t, \pi_t, T_t, B_{t-1}, L_{t-1}\}$ is the vector of variables taken as given by firms. The first order conditions associated to the firm's problem are:

$$U'\left(C_t^F\right) = \lambda_t^F \tag{44}$$

$$\lambda_t^F q_t = \beta E_t\left[\lambda_{t+1}^F\left(q_{t+1} + \frac{\partial Y_{t+1}}{\partial L_t}\right)\right] + \lambda_t^F \varphi_t m E_t\left[q_{t+1}\right] \tag{45}$$

$$(1-\varphi_t)\lambda_t^F = \beta E_t \lambda_{t+1}^F R_t \tag{46}$$

$$\xi_t = \lambda_t^F \bar{\omega}\frac{(1+\xi_V\varphi_t)}{\Phi_t} \tag{47}$$

As a result,

$$\mathcal{V}_t^F = \frac{\partial\mathcal{W}(\Omega_t^F)}{\partial N_{t-1}} = \xi_t(1-s)$$

where

$$\xi_t = \lambda_t^F \bar{\omega} \frac{(1+\xi_V \varphi_t)}{\Phi_t}$$

thus:

$$\frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} = (1-s)\frac{\bar{\omega}}{\Phi_{t+1}} \lambda_{t+1}^F (1+\xi_V \varphi_{t+1})$$

and:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s)\frac{\bar{\omega}}{\Phi_t}(1+\xi_V \varphi_t)$$

we obtain:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s)\left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta E_t\left(\frac{\lambda_{t+1}^F}{\lambda_t^F}\frac{\mathcal{V}_{t+1}^F}{\lambda_{t+1}^F}\right)\right] \tag{48}$$

hence

$$\frac{\partial\left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right)}{\partial w_t} = -(1-s)$$

Nash bargaining

$$\max_{w_t} \mathcal{S}_t = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right)^{\epsilon}\left(\frac{\mathcal{V}_t^H}{\lambda_t}\right)^{1-\epsilon}$$

The FOC is

$$\left(\frac{\mathcal{V}_t^H}{\lambda_t}\right) = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right)\frac{(1-\epsilon)(1-\Psi_t)}{\epsilon}$$

leading to

$$\epsilon\left[-\frac{\Gamma}{\lambda_t} + (w_t - b_t) + \mu E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{\mathcal{V}_{t+1}^H}{\lambda_{t+1}}\right)\right] = (1-\epsilon)\left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta E_t\left(\frac{\lambda_{t+1}^F}{\lambda_t^F}\frac{\mathcal{V}_{t+1}^F}{\lambda_{t+1}^F}\right)\right]$$

or

$$\begin{aligned} w_t \;&=\; \epsilon\left(b_t + \frac{\Gamma}{\lambda_t}\right) + (1-\epsilon)\frac{\partial Y_t}{\partial N_t} \\ &\quad + (1-\epsilon)(1-s)\bar{\omega}\beta E_t\left(\frac{\lambda_{t+1}^F}{\lambda_t^F}\frac{(1+\varphi_{t+1})}{\Phi_{t+1}}\right) \\ &\quad - (1-\epsilon)(1-s)\bar{\omega}\mu E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{(1-\Psi_{t+1})}{\Phi_{t+1}}(1+\varphi_{t+1})\right) \end{aligned} \tag{49}$$

Unlike equation (12), the Nash bargained wage, written at the steady state, depends on intertemporal consumption-savings choices through λ_t in the first term, on the RHS.

D.2.3 Calibration.

The calibration is the same as in the benchmark case (Table 1) except that Γ is computed at the steady state by using the Nash bargained wage equation and given i) the marginal utility of consumption λ (determined at the steady state by consumption choices), ii) the unemployment benefit replacement ratio (calibrated), iii) the wage (determined at the steady state by the job creation curve condition) and iv) the bargaing power (calibrated).

D.2.4 Computing welfare.

The expected lifetime utility of a worker is

$$\tilde{\mathcal{W}}^w = \mathbb{E}_0 \sum_{t=0}^{\infty} \mu^t \left[U\left(C_t\right) - N_t \Gamma\right]$$

The welfare cost of fluctuations τ is such that

$$\begin{aligned} \tilde{\mathcal{W}}^w &= \sum_{t=0}^{\infty} \mu^t \left[U\left(\bar{C}(1-\tau)\right) - \bar{N}\Gamma\right] = \frac{1}{1-\mu}\left[U\left(\bar{C}(1-\tau)\right) - \bar{N}\Gamma\right] \\ \Rightarrow \tau &= 1 - \left[\frac{[\tilde{\mathcal{W}}^w(1-\mu) + \bar{N}\Gamma](1-\sigma)}{\bar{C}^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \end{aligned}$$

We approximate welfare as

$$\begin{aligned} U\left(C_t\right) - N_t\Gamma \;\approx\; & U\left(\mathbb{E}_0 C_t\right) - \mathbb{E}_0 N_t \Gamma \\ & + U'\left(\mathbb{E}_0 C_t\right)\left(C_t - \mathbb{E}_0 C_t\right) + \frac{1}{2}U''\left(\mathbb{E}_0 C_t\right)\left(C_t - \mathbb{E}_0 C_t\right)^2 \\ & + \Gamma(N_t - \mathbb{E}_0 N_t) \end{aligned}$$

Contrary to the non-separable case, we have no covariance between C and N, and the linearity with respect to N implies that the employment variance does not matter for the welfare costs of fluctuations. What matters for the welfare is $\mathbb{E}_0[U(C_t) - N_t\Gamma]$, and thus

$$\begin{aligned}
\mathbb{E}_0[U(C_t) - N_t\Gamma] &\approx U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma + \frac{1}{2}U''(\mathbb{E}_0 C_t)\,\mathbb{E}_0[(C_t - \mathbb{E}_0 C_t)^2] \\
&\approx U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma + \frac{1}{2}U''(\mathbb{E}_0 C_t)\,[\mathbb{E}_0 C_t]^2\mathbb{E}_0[(\widehat{c}_t)^2] \\
&\approx (U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma)\left\{1 + \frac{1}{2}\frac{U''(\mathbb{E}_0 C_t)\,[\mathbb{E}_0 C_t]^2}{U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma}\mathbb{E}_0[(\widehat{c}_t)^2]\right\} \\
&\approx (U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma)\left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\}
\end{aligned}$$

where $\gamma_\Gamma = \frac{\mathbb{E}_0 N_t\Gamma}{U(\mathbb{E}_0 C_t)} < 0$ when $\sigma > 1$. Using this approximation of the instantaneous utility function, we deduce

$$\tilde{\mathcal{W}}^w \approx \frac{1}{1-\mu}(U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma)\left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\}$$

By using this last expression, we deduce that

$$\begin{aligned}
U\left(\bar{C}(1-\tau)\right) - \bar{N}\Gamma &= (U(\mathbb{E}_0 C_t) - \mathbb{E}_0 N_t\Gamma)\left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} \\
U\left(\bar{C}(1-\tau)\right) &= U(\mathbb{E}_0 C_t)\left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} - \mathbb{E}_0 N_t\Gamma\left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} + \bar{N}\Gamma \\
\frac{U\left(\bar{C}(1-\tau)\right)}{U(\mathbb{E}_0 C_t)} &= \left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} - \gamma_\Gamma\left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} + \frac{\bar{N}}{\mathbb{E}_0 N_t}\gamma_\Gamma \\
\frac{U\left(\bar{C}(1-\tau)\right)}{U(\mathbb{E}_0 C_t)} &= \left\{1 - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} - \gamma_\Gamma\left\{1 - \frac{\bar{N}}{\mathbb{E}_0 N_t} - \frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} \\
\frac{U\left(\bar{C}(1-\tau)\right)}{U(\mathbb{E}_0 C_t)} &= \left\{1 - \gamma_\Gamma + \gamma_\Gamma\frac{\bar{N}}{\mathbb{E}_0 N_t} - (1-\gamma_\Gamma)\frac{1}{2}\frac{\sigma(1-\sigma)}{1-\gamma_\Gamma}Var(\widehat{c}_t)\right\} \\
\frac{U\left(\bar{C}(1-\tau)\right)}{U(\mathbb{E}_0 C_t)} &= \left(1 + \gamma_\Gamma\frac{\bar{N} - \mathbb{E}_0 N_t}{\mathbb{E}_0 N_t} - \frac{1}{2}\sigma(1-\sigma)Var(\widehat{c}_t)\right) \\
\Rightarrow 1-\tau &= \left(\frac{\mathbb{E}_0 C}{\bar{C}}\right)\left(1 + \gamma_\Gamma\frac{\bar{N} - \mathbb{E}_0 N_t}{\mathbb{E}_0 N_t} - \frac{1}{2}\sigma(1-\sigma)Var(\widehat{c}_t)\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

D.3 Financial shocks

As in Liu et al. (2013) and Jermann & Quadrini (2012), the financial shock is captured as a shock on m. This is interpreted as shocks on the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. The financial shock follows the stochastic process

$$log(m_t) = (1-\rho_m)log(\overline{m}) + \rho_m log(m_{t-1}) + \epsilon_t^m$$

with the calibrated values from Liu et al. (2013)'s estimation results ($\rho_m = 0.9804$ and $\sigma^{\epsilon^m} = 0.0112$). The standard deviation of technological innovation σ_A is adjusted to match the observed standard deviation of output. This calibration is used in column (2) Table 5.

D.4 Model with capital

Model with capital as collateral. In the model with capital households' behaviorS do not change. The introduction of capital alters the entrepreneurs' problem. As in Liu et al. (2013), the production function is now

$$Y_t = A_t \left[L_t^{\phi} K_{t-1}^{1-\phi} \right]^{1-\alpha} N_t^{\alpha}$$

with K the stock of capital. Capital accumulation is subject to adjustment costs such that

$$K_t = (1-\delta) K_{t-1} + I_t + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \quad \left(\lambda_t^K\right)$$

with I_t the investment flow and Ω the scale parameter on adjustment costs. The entrepreneur's budget constraint is now

$$C_t^F + R_{t-1}B_{t-1} + q_t^k \left[K_t - (1-\delta) K_{t-1}\right] + q_t \left[L_t - L_{t-1}\right] + w_t N_t + \bar{\omega} V_t \leq Y_t + B_t + \pi_t \quad (\lambda_t) \tag{50}$$

with λ_t the Lagrange multiplier on equation (50) and q_t^k the price of capital in consumption units. The collateral constraint now includes capital

$$B_t + \omega V_t \leq m \left(\mathbb{E}_t \left[q_{t+1}^k \right] K_t + \mathbb{E}_t \left[q_{t+1} \right] L_t \right) \tag{51}$$

The firm's program is

$$\mathcal{W}(\Omega_t^F) = \max_{C_t^F, L_t, B_t, V_t, N_t, K_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t \left[\mathcal{W}(\Omega_{t+1}^F)\right] \right\} \tag{52}$$

$$s.t. \quad \begin{cases} -C_t^F - R_{t-1}B_{t-1} - q_t\left[L_t - L_{t-1}\right] - w_t N_t - \bar{\omega} V_t & \\ -q_t^k \left[K_t - (1-\delta) K_{t-1}\right] + Y_t\left(A_t, K_{t-1}, L_t, N_t\right) + B_t = 0 & (\lambda_t^F) \\ -B_t - \bar{\omega} V_t + m\mathbb{E}_t\left[q_{t+1}^k\right] K_t + m\mathbb{E}_t\left[q_{t+1}\right] L_t = 0 & (\lambda_t^F \varphi_t) \\ -N_t + (1-s)N_{t-1} + \Phi_t V_t = 0 & (\xi_t) \\ -K_t + (1-\delta) K_{t-1} + I_t + \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t = 0 & (\lambda_t^K) \end{cases}$$

Let us define the shadow price of capital in consumption units

$$q_t^k = \frac{\lambda_t^K}{\lambda_t^F}$$

then the FOCs with respect to K_t is

$$q_t^k = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} \left((1-\alpha)(1-\phi) \frac{Y_{t+1}}{K_t} + q_{t+1}^k (1-\delta) \right) \right] + \varphi_t m \mathbb{E}_t \left[q_{t+1}^k\right]$$

and the choice of investment is such that

$$1 = q_t^k \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \frac{I_t}{I_{t-1}}\Omega\left(\frac{I_t}{I_{t-1}} - 1\right) \right] + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} q_{t+1}^k \Omega \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right]$$

Calibration. The capital adjustment cost parameter $\Omega = 0.1881$ and $\phi = 0.0695$ are set to the estimated values found in Liu et al. (2013). $\delta = 0.025$ as in Liu et al. (2013). $\alpha = 0.805$ is set so as to mimic the capital-output ratio (4.15 in the US data, as reported in Liu et al. (2013)). $m = 0.123$ is chosen to match the benchmark value of $B/Y = 0.59$. Finally, the standard deviation of technological innovation σ_A is adjusted to match the observed standard deviation of output. This calibration is used in columns (3) and (5) of Table 5.

D.5 Model with capital as collateral with wage and investment costs in the financial constraint

Model. The behaviors of the households do not change. As in Millard et al. (2017), we introduce wage payment and investment cost in the borrowing constraint. The financial constraint becomes

$$B_t + \omega V_t + w_t N_t + I_t \leq m\left(\mathbb{E}_t\left[q^k_{t+1}\right] K_t + \mathbb{E}_t\left[q_{t+1}\right] L_t\right) \tag{53}$$

The firm's program is

$$\begin{aligned}
\mathcal{W}(\Omega^F_t) &= \max_{C^F_t, L_t, B_t, V_t, N_t, K_t} \left\{U\left(C^F_t\right) + \beta E_t\left[\mathcal{W}(\Omega^F_{t+1})\right]\right\} \qquad (54)\\
s.t. & \left\{\begin{array}{rcll}
-C^F_t - R_{t-1}B_{t-1} - q^k_t\left[K_t - (1-\delta)K_{t-1}\right] - q_t\left[L_t - L_{t-1}\right] - w_t N_t - \bar{\omega}V_t & & & \\
+Y_t\left(A_t, K_{t-1}, L_t, N_t\right) + B_t & = & 0 & (\lambda^F_t) \\
-\mathbf{q}^k_t\left[K_t - (1-\delta)K_{t-1}\right] - B_t - \bar{\omega}V_t - w_t N_t + mE_t\left[q^k_{t+1}\right]K_t + mE_t\left[q_{t+1}\right]L_t & = & 0 & (\lambda^F_t\varphi_t) \\
-N_t + (1-s)N_{t-1} + \Phi_t V_t & = & 0 & (\xi_t)
\end{array}\right.
\end{aligned}$$

$$N_t \;:\; \lambda^F_t\left(\frac{\partial Y_t}{\partial N_t} - w_t - \varphi_t w_t\right) + \beta E_t \xi_{t+1}(1-s) = \xi_t$$

$$\begin{aligned}
V_t : \frac{\lambda^F_t\left(\bar{\omega} + \bar{\omega}\varphi_t\right)}{\Phi_t} &= \xi_t \text{so that} \\
E_t \frac{\lambda^F_{t+1}\left(\bar{\omega} + \bar{\omega}\varphi_{t+1}\right)}{\Phi_{t+1}} &= E_t\xi_{t+1}
\end{aligned}$$

We then get the job creation condition

$$\bar{\omega}\frac{(1+\varphi_t)}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t\left(1+\varphi_t\right) + (1-s)\beta\frac{\bar{\omega}}{\lambda^F_t}E_t\left[\lambda^F_{t+1}\frac{(1+\varphi_{t+1})}{\Phi_{t+1}}\right]$$

FOCs with respect to K_t Let us define the shadow price of capital in consumption units $q^k_t = \frac{\lambda^K_t}{\lambda^F_t}$. The FOC is

$$q^k_t\left(1+\varphi_t\right) = \beta E_t\left[\frac{\lambda^F_{t+1}}{\lambda^F_t}\left(\left(1-\alpha\right)\left(1-\phi\right)\frac{Y_{t+1}}{K_t} + q^k_{t+1}\left(1-\delta\right)\right)\right] + \varphi_t m E_t\left[q^k_{t+1}\right] \tag{55}$$

Without financial frictions, $\varphi_t = 0$, we recover the usual FOC on capital.

FOCs with respect to I_t

$$(1+\varphi_t) = q_t^k \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \frac{I_t}{I_{t-1}}\Omega\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta E_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} q_{t+1}^k \Omega \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{56}$$

Without financial frictions, $\varphi_t = 0$, we recover the usual FOC on investment.

Calibration. We consider the same calibration as in section D.4, except for $m = 0.28$ that is adjusted to match the benchmark value of $B/Y = 0.59$. $\alpha = 0.81$ is set so as to mimic the capital-output ratio (4.15 in the US data, as reported in Liu et al. (2013)). The standard deviation of technological innovation σ_A is adjusted to match the observed standard deviation of output. This calibration is used in column (4) of Table 5.

Table 5: Business-cycle volatility : Models versus data

	(0) Data		(1) Benchmark		(2) Financial shocks		(3) Capital		(4) Financial shocks and capital	
σ_A			0.0031		0.0024		0.0066		0.0067	
	std(.)		std(.)		std(.)		std(.)		std(.)	
Y	1.44	**	1.44	**	1.44	**	1.44	**	1.44	**
C	0.81	*	0.88	*	0.85	*	0.80	*	0.82	*
N	0.72	*	0.74	*	0.86	*	0.65	*	0.65	*
Y/N	0.54	*	0.28	*	0.21	*	0.52	*	0.52	*
w	0.62	*	0.49	*	0.50	*	0.35	*	0.36	*
U	7.90	*	5.45	*	6.30	*	4.81	*	4.80	*
Ψ	5.46	*	6.26	*	7.50	*	12.2	*	12.5	*
V	9.96	*	12.7	*	15.5	*	24.1	*	24.5	*
B	1.68	*	1.35	*	1.63	*	0.27	*	1.03	*
q	3.21	*	2.59	*	2.83	*	0.53	*	0.89	*
I	4.59	*		*		*	2.52	*	2.96	*
$corr(U,\Psi)$	-0.91		-0.86		-0.86		-0.54		-0.54	
$corr(U,V)$	-0.97		-0.71		-0.72		-0.40		-0.39	

** std (in percentage); * relative to GDP std

E Understanding the welfare effect of financial shocks using Mickey Mouse models

In this section, we show in Mickey Mouse models that financial shocks do decrease welfare costs of fluctuations in a model with land only (section E.1). This provides a rationale for the quantitative results found in column (5) in Table 4. In contrast, financial shocks do increase business-cycle costs in a model with capital (section E.2), which explains the quantitative results found in column (7) in Table 4.

E.1 A simple model with land : Financial shocks reduce welfare costs

E.1.1 Household

The dynamic problem of a typical household can be written as follows

$$\mathcal{W}(\Omega_t^H) = \max_{C_t, B_t} \left\{ U(C_t) + \mu \mathbb{E}_t \mathcal{W}(\Omega_{t+1}^H) \right\} \quad \text{s.c.} \quad C_t + B_t \leq R_{t-1} B_{t-1} + w_t$$

given the initial conditions on state variables B_0 and $\Omega_t^H = \{w_t, B_{t-1}\}$, the vector of variables taken as given by households. Let λ_t be the shadow price of the budget constraint. The FOC are $(C_t)^{-\sigma} = \lambda_t$ and $-\lambda_t + \mu \mathbb{E}_t [R_t \lambda_{t+1}] = 0$. The labor supply is $N_t = 1$.

E.1.2 Entrepreneur

The firm's program is

$$\begin{array}{rcl} \mathcal{W}(\Omega_t^F) & = & \max\limits_{C_t^F, L_t, B_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t \left[\mathcal{W}(\Omega_{t+1}^F) \right] \right\} \\ s.t. & & \left\{ \begin{array}{rcll} -C_t^F - R_{t-1}B_{t-1} - q_t \left[L_t - L_{t-1}\right] - w_t N_t + Y_t\left(A_t, L_{t-1}, N_t\right) + B_t & = & 0 & (\lambda_t^F) \\ -B_t + m_t \mathbb{E}_t \left[q_{t+1} L_t\right] & = & 0 & (\lambda_t^F \varphi_t) \end{array} \right. \end{array}$$

given the initial conditions B_0, where $\Omega_t^F = \{w_t, B_{t-1}, L_{t-1}, m_t\}$ is the vector of variables taken as given by firms. Letting λ_t^F and $\lambda_t^F \varphi_t$ be the Lagrange multipliers associated to each constraint, the first order conditions of problem read:

$$\begin{aligned}
U'\left(C_t^F\right) &= \lambda_t^F \\
\lambda_t^F q_t &= \beta\mathbb{E}_t\left[\lambda_{t+1}^F\left(q_{t+1}+\frac{\partial Y_{t+1}}{\partial L_t}\right)\right]+\lambda_t^F\varphi_t m_t\mathbb{E}_t\left[q_{t+1}\right] \\
(1-\varphi_t)\lambda_t^F &= \beta\mathbb{E}_t\lambda_{t+1}^F R_t \\
w_t &= \frac{\partial Y_t}{\partial N_t}
\end{aligned}$$

E.1.3 Equilibrium

Given that $N_t = 1$ and $L_t = 1\ \forall t$ and assuming that $Y_t = A_t L_t^{1-\alpha} N_t^{\alpha} = A_t$, we deduce

Dynamic system	Steady state
$1 = \mu\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right]R_t$	$1 = \mu R$
$q_t = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}^F}{\lambda_t^F}\left(q_{t+1}+(1-\alpha)A_{t+1}\right)\right]+\varphi_t m_t\mathbb{E}_t\left[q_{t+1}\right]$	$q = \frac{\beta(1-\alpha)}{1-\beta-\varphi m}A$
$1 = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}^F}{\lambda_t^F}\right]R_t+\varphi_t$	$1 = \frac{\beta}{\mu}+\varphi$
$w_t = \alpha A_t$	$w = \alpha A$
$C_t + B_t = R_{t-1}B_{t-1}+w_t$	$C = \frac{1-\mu}{\mu}B+\alpha A$
$A_t + B_t = C_t^F + R_{t-1}B_{t-1}+w_t$	$C^F = (1-\alpha)A-\frac{1-\mu}{\mu}B$
$B_t = m_t\mathbb{E}_t\left[q_{t+1}\right]$	$B = mq$

How does B change with m? This depends on $mq = \frac{m\beta(1-\alpha)}{1-\beta-\varphi m}A \equiv F(m,A)$. We have $F_m' = \frac{\beta(1-\alpha)(1-\beta)}{(1-\beta-\varphi m)^2} > 0$ whereas $F_{mm}'' = \frac{2\varphi\beta(1-\alpha)(1-\beta)}{(1-\beta-\varphi m)^3} > 0$. Hence, mq is convex implying that $E[mq] > \overline{mq}$. Thus, expected debt is larger than steady-state debt: $E[B] > \overline{B}$. Given the worker's budget constraint ($C = \frac{1-\mu}{\mu}B+\alpha A$), we have $E[C] > \overline{C}$ and expected consumption is larger than steady-state consumption. The level effect on consumption actually implies that financial shocks in an economy with land are actually welfare-improving. Uncertainty generates a premium on the price of land as its return is risky for the bank. In this case, the value of worker's wealth increases on average, which explains the larger consumption in the stochastic economy, with respect to the stabilized economy.

How does B change with A? Given that F is linear in A, we deduce that $E[mq] = \overline{mq}$, hence $E[B] = \overline{B}$ and thus $E[C] = \overline{C}$. Technological shocks are neutral on the level effect of the welfare costs of the business cycle. This last result shows that welfare costs provided by our benchmark model are the result of the interaction between labor market and financial frictions. Indeed, without labor market frictions, the technological shocks are not costly in an economy with only financial constraints.

E.2 A simple model with capital : Financial shocks increase welfare costs

E.2.1 Individual behaviors

The household's program does not change. The firm's program becomes

$$\begin{array}{rcl} \mathcal{W}(\Omega_t^F) & = & \max_{C_t^F, B_t, K_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t \left[\mathcal{W}(\Omega_{t+1}^F)\right] \right\} \\ s.t. & & \left\{ \begin{array}{rcll} -C_t^F - R_{t-1}B_{t-1} - w_t N_t - K_t + A_t K_{t-1}^{1-\alpha} N_t^{\alpha} + B_t & = & 0 & (\lambda_t^F) \\ -B_t + m_t K_t & = & 0 & (\lambda_t^F \varphi_t) \end{array} \right. \end{array}$$

The FOCs are

$$\begin{array}{rcl} U'\left(C_t^F\right) & = & \lambda_t^F \\ \lambda_t^F & = & \beta \mathbb{E}_t \left[\lambda_{t+1}^F \dfrac{\partial Y_{t+1}}{\partial K_t} \right] + \lambda_t^F \varphi_t m_t \\ (1-\varphi_t)\lambda_t^F & = & \beta \mathbb{E}_t \lambda_{t+1}^F R_t \\ w_t & = & \dfrac{\partial Y_t}{\partial N_t} \end{array}$$

E.2.2 Equilibrium

Given that $N_t = 1 \; \forall t$, we deduce

Dynamic system	Steady state
$1 = \mu \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\right] R_t$	$1 = \mu R$
$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F}(1-\alpha)A_{t+1}K_t^{-\alpha}\right] + \varphi_t m_t$	$K = \left(\frac{\beta(1-\alpha)}{1-\varphi m} A\right)^{\frac{1}{\alpha}}$
$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F}\right] R_t + \varphi_t$	$1 = \frac{\beta}{\mu} + \varphi$
$w_t = \alpha A_t K_t^{1-\alpha}$	$w = \alpha A K^{1-\alpha}$
$C_t + B_t = R_{t-1}B_{t-1} + w_t$	$C = \frac{1-\mu}{\mu} B + \alpha A K^{1-\alpha}$
$A_t K_{t-1}^{1-\alpha} + B_t = C_t^F + R_{t-1}B_{t-1} + K_t + w_t$	$C^F = (1-\alpha)AK^{1-\alpha} - \frac{1-\mu}{\mu} B - K$
$B_t = m_t K_t$	$B = mK$

The steady state, conditional to $\{A, m\}$, gives $K = \mathcal{K}(m, A) = (\beta(1-\alpha)A)^{\frac{1}{\alpha}}(1-\varphi m)^{-\frac{1}{\alpha}}$ and $B = \mathcal{B}(m, A) = m\mathcal{K}(m, A)$. Assume a first restriction, which is satisfied in our calibration exercises, namely $C^F \approx 0$ ie. $\frac{1-\mu}{\mu}B \approx (1-\alpha)AK^{1-\alpha} - K$. We deduce that consumption C is given by $C \approx A\mathcal{K}(m, A)^{1-\alpha} - \mathcal{K}(m, A)$,

$$\text{with} \quad \begin{cases} \mathcal{K}'_m(m, A) &= \frac{1}{\alpha}\frac{\varphi}{1-\varphi m}\mathcal{K}(m, A) > 0 \\ \mathcal{K}''_{mm}(m, A) &= \frac{1+\alpha}{\alpha}\frac{\varphi}{1-\varphi m}\mathcal{K}'_m(m, A) > 0 \end{cases}$$

This leads to

$$C'' = \mathcal{K}'_m(m, A)\frac{\varphi}{1-\varphi m}\frac{1}{\alpha}\left[(1-\alpha)A\mathcal{K}(m, A)^{-\alpha} - (1+\alpha)\right]$$

Given that $\mathcal{K}'_m(m, A)\frac{\varphi}{1-\varphi m}\frac{1}{\alpha} > 0$, C'' has the same sign as the term between brackets, which consists of 2 terms. The first term $(1-\alpha)A\mathcal{K}(m, A)^{-\alpha} < 1$ because it represents an interest rate, and the second term $1 + \alpha > 1$. This implies that $C'' < 0$, hence $E[C] < \overline{C}$ when the uncertainty comes from financial shocks. This shows that fluctuations in m reduce welfare.

Why do changes in m increase welfare costs when the collateral includes capital? In presence of financial shocks, the return on the collateral becomes risky for banks. This uncertainty generates a premium on the borrowing constraint (an over-accumulation). This generates a new motive to increase leveraging (B increases in the volatile economy relative to the stabilized economy) and thus, capital. As capital in our economy is characterized by decreasing returns to scale, this over-accumulation is then costly in terms of consumption.

F Asymmetric welfare costs of business cycles

F.1 Benchmark

As stressed by Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2017), search and matching models feature asymmetric responses to business-cycle shocks; recessions (expansions) are characterized by severe and rapid rises (gradual decline) in unemployment. These asymmetric fluctuations are supported by empirical evidence.[53] The IRFs in model B (without financial frictions) are consistent with these features. Figure 2, panel (b), displays the employment response to the calibrated positive productivity shock when

[53]See e.g. MacKay & Reis (2008) and Petrosky-Nadeau & Zhang (2017), Ferraro (2018) or Adjemian et al. (2016).

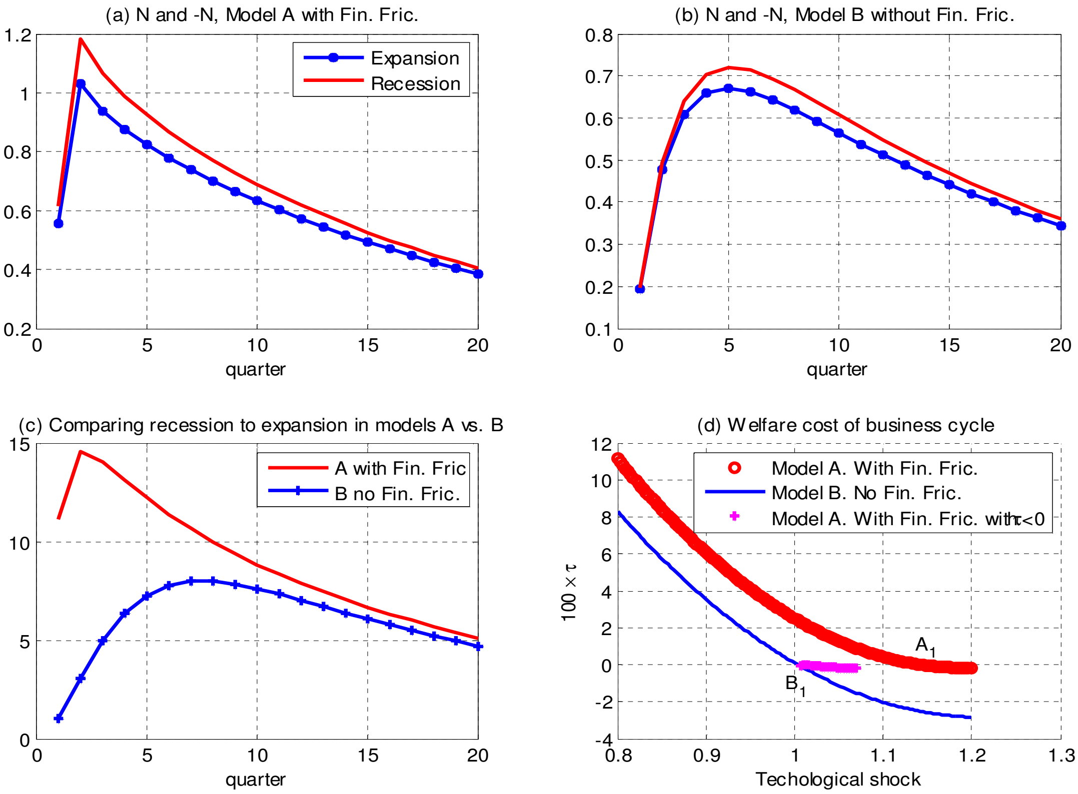
the economy is in boom – i.e., starting from a point where the economy is already hit by the same shock (dotted line)– and to a negative shock of the same magnitude when the economy is in recession (solid line). Employment falls more in a recession than it increases in a boom. Figure 2, panel (a), shows the IRFs from model A (with financial frictions) in a recession versus economic boom. The asymmetric response of employment is even larger than in model B. In order to measure this increased asymmetry due to the presence of financial frictions, we report, in panel (c), the gaps between IRFs in recession versus boom in both models. The gap between IRFs is twice as large in the economy with financial frictions (model A) as in the economy without financial frictions (model B); the maximum gap is around 15% in model A, but only 7.5% in model B.[54]. In addition, since labor market adjustments are connected to credit market conditions, the immediate response of credit market conditions directly affect employment dynamics. The maximum gap is reached after two quarters in model A and seven quarters in model B.

In order to measure the implied business-cycle costs of these non-linearities, we compute the time-varying welfare cost τ as in Petrosky-Nadeau & Zhang (2017) for model A (with financial frictions) and model B (without financial frictions).[55] Figure 2, panel (d), plots the welfare cost $\tau \times 100$ against the technological shock. First, in model B, the welfare cost is countercyclical. In addition, welfare gains in expansion are much lower than welfare costs in recession, which is consistent with the asymmetric responses to business-cycle shocks displayed in matching models. Starting from point B_1, a 10% increase in productivity (from 1 to 1.1) results in a welfare gain of 2% while, for a fall in productivity of the same magnitude (from 1 to 0.9), the welfare loss is twice as large. Interestingly, welfare costs in model A are all located above the corresponding costs for model B, suggesting that the presence of financial frictions moves the economy to regions with higher unemployment levels and therefore higher welfare costs. Furthermore, since the welfare cost function shifts upwards, the business cycle starts being desirable ($\tau < 0$) only for large expansions (for a technological change of at least 1.15, beyond point A_1 in Figure 2). Even in the case of large expansions, the slope of the welfare cost function is nearly flat; welfare gains remain very small. In that sense, we argue that the presence of financial frictions makes the welfare costs more asymmetric than in the matching model. To illustrate this point, we report, in panel (d) of Figure 2, the welfare

[54]This last case summarizes the results reported in Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2017).

[55]The welfare cost at each date is based on a comparison between the deterministic worker's welfare and the fluctuating worker's welfare summarized by his value function. This value function is time-varying. Its expected value is computed, for each current state of the economy, using the second-order Dynare approximation of endogenous variables (see Adjemian et al. (2014)). We consider the decision rules around the mean, rather than the steady state, which is the default setting in Dynare

Figure 2: The Welfare cost of business cycles and asymmetry: Why do financial frictions matter?



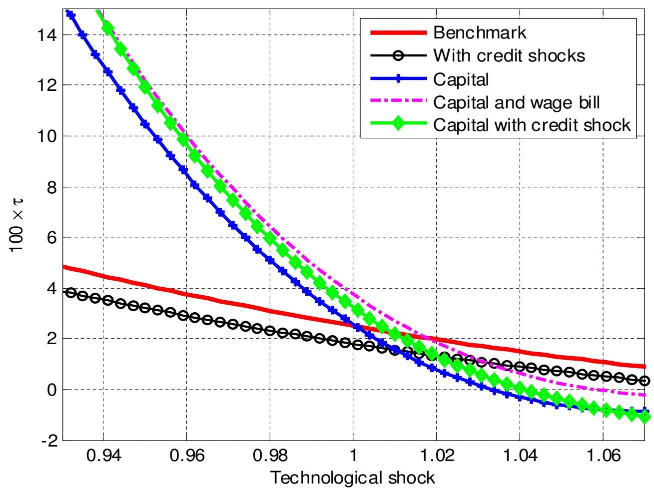
(a): IRF of Employment ×100, N in response to a positive productivity shock starting from expansion (dotted line) and $-N$ in response to a negative productivity shock starting from recession (solid line) in model A with financial frictions. The size of the shock is calibrated as above. (b): IRF of Employment ×100, N in response to a positive productivity shock starting from expansion (dotted line) and $-N$ in response to a negative productivity shock starting from recession (solid line) in model B without financial frictions. The size of the shock is calibrated as above. (c) : $\frac{-IRF_{recession}-IRF_{expansion}}{IRF_{expansion}} \times 100$ in models A (solid line) and B (dotted line). (d) The steady-state value of technological shock is 1. Curves are quadratic fit over 30,000 simulations for model A (circle)and model B (solid line). $+$: segment of cost function for model A (only segment with $\tau < 0$, beyond point A_1) s hifted to point B_1.

cost function of model A (just the segment beyond point A_1, with $\tau < 0$) at point B_1. It can then be easily seen that, when both models predict welfare gains (expansions), welfare gains remain small in model A, with financial frictions, while they increase quasi-linearly without financial frictions in model B.

F.2 Robustness

Figure 3 also shows that the asymmetries intrinsic to our model are not affected by the introduction of an additional shock. Indeed, the structure of our framework remains unaffected.

Figure 3: The Welfare cost of business cycles and asymmetry



The steady-state value of technological shock is 1. Curves are quadratic fit over 30,000 simulations for each model.