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I. ILIOPULOS, F. LANGOT and T. SOPRASEUTH

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Université d'Evry Val d'Essonne, 4 bd. F. Mitterrand, 91025 Evry CEDEX

Welfare Cost of Fluctuations When Labor Market Search Interacts with Financial Frictions

Eleni Iliopulos* François Langot† Thepthida Sopraseuth‡§

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Abstract

We study the welfare costs of business cycles in a search and matching model with financial frictions à la Kiyotaki & Moore (1997). We investigate the mechanisms that allow the model to replicate the volatility on labor *and* financial markets. Business cycle costs are sizable and asymmetric. This result is not trivial because the introduction of financial frictions does not *per se* always dampen welfare. Indeed, when credit costs are counter-cyclical and very responsive to productivity shocks, firms could benefit so much from the fall in hiring costs that the average job finding rate lies above its steady state value.

JEL Classification : E32, J64, G21

Keywords: Welfare, business cycle, financial friction, labor market search

*U. of Evry and CEPREMAP. `eleni.iliopulos@univ-evry.fr`

†U. of Le Mans (GAINS-TEPP & IRA), Paris School of Economics and IZA. `flangot@univ-lemans.fr`

‡U. of Cergy Pontoise (THEMA) and CEPREMAP. `thepthida.sopraseuth@u-cergy.fr`

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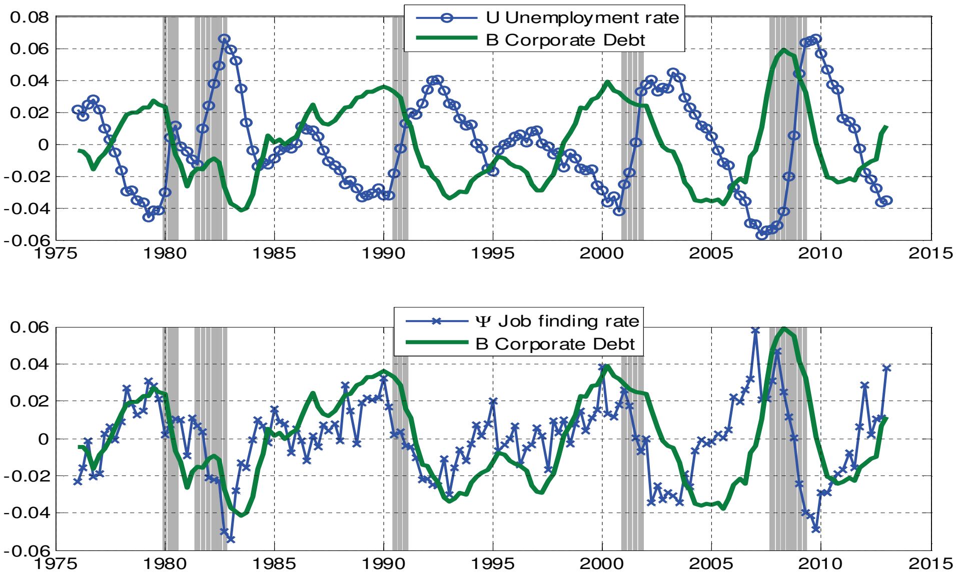
1 Introduction

This paper investigates the welfare costs of fluctuations in an original framework that features strong interactions between financial and labor market frictions. A glance at the data suggests the empirical relevance of this interaction. Figure 1 shows that episodes of job creation (when the job-finding rate Ψ rises) are also times when firms accumulate debt.[1]

Figure 1: Cyclicality of unemployment rate U, job finding rate Ψ and debt stock B.



HP Filtered logged quarterly data. Shaded area shows recessions (NBER dates). HP filtered data on U and Ψ were divided by 4 for the purpose of scale consistency. Smoothing parameter: 1600. Source : See Appendix A.

A satisfactory assessment of welfare costs of the business cycle must rely on a model that replicates the volatility of aggregates (quantities and prices) on both labor and financial markets.[2] We build a search-and-matching DSGE model, following Mortensen & Pissarides (1994) (hereafter DMP). Entrepreneurs' access to credit is limited by a collateral constraint as in Kiyotaki & Moore (1997) because of enforcement limits. Entrepreneurs borrow so as to finance intra-period hiring costs, which gives rise to the interaction between labor market and financial frictions. With respect to the existing literature, we pay attention to the empirical relevance of the model with respect to *both* labor and financial markets. Moreover, we study

[1]Jermann & Quadrini (2012) also notice that debt repurchases (a reduction in outstanding debt) increase during or around recessions.

[2]On the labor market, we focus on unemployment and wage, whereas on the financial market, we focus on corporate debt and land price, considered as the collateral for the borrowing contract. Chéron & Langot (2004) and Pissarides (2009) echo the old Keynes-Tarshis-Dunlop controversy by stressing the need to understand fluctuations in both unemployment and wages. Since the work of Shimer (2005), real-wage rigidity is known to solve the volatility puzzle in the Diamond-Mortensen-Pissarides model. However, the extreme view of a fixed wage is rejected by the data.

the mechanisms that allow our benchmark model to replicate the volatilities of financial and labor prices and aggregates. The introduction of financial frictions is not *per se* sufficient to lead a DMP model to generate volatile labor and financial variables *and* sizeable welfare costs. We thus analyze the specific features of our framework at the roots of our results: i) volatile and data-consistent labor and financial markets; ii) large welfare costs associated to fluctuations.[3,4]

The economic mechanisms at work are the following. On the labor market, the average time to fill a vacancy increases during booms (due to the congestion effect inherent to DMP models). This raises the quantity of credit needed to fund the hiring policy. Firms need to get more indebted, which is costly. On the other hand, during booms, the rise in the price of assets (i.e., firms' collateral) tends to relax the borrowing constraint and the cost of credit decreases. In our model, financial frictions are such that firms are reluctant to accept higher wages during booms. This countercyclical force tends to dampen wage increases so as to preserve firms' incentive to open new vacancies. The model also correctly predicts the volatility on the financial market. Indeed, as land supply does not vary over the business cycle, any change in land demand results in changes in the real-estate price. The model is then able to generate volatile collateral price (about 3 times more volatile than output). The predicted debt volatility is consistent with the data (about 1.5 times more volatile than output). This is due to the interaction between financial and labor market frictions in the borrowing constraint. Indeed, absent labor-market frictions in the borrowing constraint, the volatility of the collateral price directly predicts the volatility of corporate debt. In this case, the model would predict a debt volatility that is too large with respect to data. When vacancies are incorporated into the borrowing constraint, the variance of the collateral price equals the sum of the variance of debt, that of vacancies and the covariance between debt and vacancies. In spite of a large volatility of the collateral price, because vacancies are very volatile and covariate positively with debt, our model is able to match the volatility of debt.

Finally, notice that there is only a technological shock in our model.[5] This restrictive view of the sources of fluctuations has a double advantage: first, it allows us to isolate the mechanism

[3]One could argue that wage rigidity is enough to solve the difficult exercise of obtaining a volatile labor-market response to productivity shocks so as to entail to sizeable welfare costs (see Hairault et al. (2010)). We argue that wage rigidity is actually not a relevant solution. First, wage volatility in US data is 60% that of output (which points out that wages do respond to the business cycle, contrary from the rigid wage assumption). Secondly, we show that, with rigid wages, financial frictions can even lead to welfare *gains* from fluctuations. Hence, wage flexibility founded on the Nash-bargaining rule plays a crucial in the understanding of sizeable welfare costs of fluctuations in an economy with financial and labor market frictions. This allows the model to be potentially consistent with the realistic volatilities and welfare costs of fluctuations.

[4]In order to show that this result is robust, we propose several extensions of our basic model in section 5.4.

[5]We include financial shocks as a robustness check in section 5.4.

at work in the model; second, it facilitates the comparisons of our framework with the other contributions on the Shimer (2005) puzzle.[6]

We show how financial frictions raise welfare costs of fluctuations several times those with only labor frictions. This is not a trivial result as financial frictions do not *per se* dampen welfare. Indeed, by increasing the sensitivity of the hiring policy to the shocks, these financial frictions can potentially generate welfare *gains* from fluctuations: when credit cost is very responsive to the productivity shock, firms benefit so much from the falling recruitment costs in expansion that the average job finding rate lies above its steady state value, thereby reducing welfare costs of fluctuations.

With respect to a DSGE model with labor market search but without financial frictions, the relative volatility of the job-finding rate is multiplied by two. The welfare cost of the business-cycle in a economy with financial and labor market frictions is 2.5% of workers' permanent consumption. It drastically falls without financial frictions and labor market frictions only (at 0.12% for workers). These costs are far larger than the estimates by Lucas (1987, 2003), who reports a welfare cost of 0.05% in the case of logarithmic utility. We compute the time-varying welfare cost and report its empirical distribution for the model with financial and search frictions and the model with search frictions only. Whatever the model, welfare costs in recessions are larger than welfare gains in expansion. This result captures the asymmetric business cycles in our nonlinear environment. In addition, the presence of financial frictions shifts the distribution of welfare costs to regions showing more asymmetry and greater losses; not only are welfare costs larger whatever the state of the economy (welfare gains therefore appear only in case of large expansions), but welfare gains remain small in case of expansion (while they increase quasi-linearly without financial frictions).

The paper is organized as follows. Section 2 outlines our original contribution to the literature, and section 3 describes the model. Section 4 provides the analysis on the interaction between financial and labor market frictions. Our quantitative analysis is developed in section 5. Section 6 concludes the paper.

2 Related literature

Our contribution lies in bridging the gap between two strands of the literature, one studying the welfare costs of fluctuations and the other investigating the macroeconomic impact of

[6] This approach differs from the method proposed by Liu, Wang & Zha (2013) and Christiano et al. (2016), but shares its feature of the small sample size.

financial frictions on labor market dynamics.

Lucas (1987, 2003) shows that welfare costs associated with business cycles are negligible. In his framework, costs associated with recessions are indeed compensated by the gains during booms. In this paper, we challenge Lucas's result by proposing that a nonlinear DSGE model, where the allocation is sub-optimal, can generate significant asymmetries at business-cycle frequency. In our model, the costs of recessions cannot be compensated by the gains of expansions because average employment and average consumption are significantly lower than their deterministic steady-state levels. Welfare costs of fluctuations can then be significantly greater than those found by Lucas.[7] As explained by Hairault et al. (2010), Jung & Kuester (2011), and Petrosky-Nadeau & Zhang (2013), canonical search-and-matching labor frictions introduce a gap between the deterministic steady-state and mean unemployment levels. Using the DMP model, they show how this generates business-cycle costs ranging between 0.2% and 1.2% of permanent consumption in their calibrated models. However, these works neglect financial frictions.

In this paper, we study the interaction between nonlinearities that are associated with the matching process on the labor market and that are specific to financial frictions so as to focus on their impact on welfare throughout the cycle. This analysis constitutes our original contribution to the literature on the welfare costs of fluctuations.[8] In our model, the financial contract concerns both firms (i.e., the borrowers) and workers (i.e., the lenders). As suggested by Pissarides (2011), the "*equilibrium matching models are built on the assumption of perfect capital markets. ... But future work needs to explore other assumptions about capital markets, and integrate the financial sector with the labour market. This might suggest another amplification mechanism for shocks, independently from wage stickiness or fixed costs*" (Pissarides (2011)).[9] The link between the matching model and financial imperfections is in fact intuitive. Search activity is costly for firms and needs to be financed with borrowing. However, this investment cannot be used as a collateral.[10] Our paper also complement the view developed by Hall (2017): unemployment increases in periods of high discount rates. In our framework, it is not the absolute value of the discount rate that matters but the gap between borrowers' (entrepreneurs) and lenders' (households) discount rates. Our gen-

[7] In case of a significant gap between average and deterministic steady-state values, such costs are of a first-order magnitude – as the costs of tax distortion also evaluated by Lucas (1987).

[8] This study differs from previous works on business-cycle costs, such as Beaudry & Pages (2001), Storesletten et al. (2001) or Krebs (2003), in that it regards interactions between the labor and financial frictions as the root of the welfare cost of business cycles.

[9] Dromel et al. (2010) show that Pissarides (2011)' intuition is supported by significant empirical links between unemployment dynamics and financial frictions.

[10] In addition, financial frictions may induce high nonlinearities because of borrowing constraints, which can amplify welfare costs of the business cycle.

eral equilibrium approach also explains the dynamics of this gap (the financial wedge) in a framework where fluctuations are driven by a technological shock.[11]

We thus depart from the DMP models, too, where the borrowing constraint is supported by households, with access to incomplete financial markets. Indeed, Krusell et al. (2010) show that the nonlinearities linked to financial constraints à la Aiyagari (1994) do not change the gaps of endogenous variables between bad and good states, in contrast to linear-utility models where this type of financial constraints does not matter.[12]

Recent research has studied how evolving conditions on credit markets affect the dynamics of labor markets and improve the ability of standard search models to match data.[13] We contribute to this literature by using a streamlined model à la Kiyotaki & Moore (1997). We show that the interaction between financial and credit frictions does not systematically generate a strong propagation mechanism. We show why in our framework financial frictions generate an amplification effect, which depends both on a credit-multiplier effect[14] and an additional amplification mechanism. The latter mechanism affects the wage-bargaining process and is specific to financial frictions *à la* Kiyotaki & Moore (1997). Our quantitative results suggest that these mechanisms are sufficient to solve the volatility puzzle of the DMP model. Therefore, it is not necessary to combine exogenous fluctuations in collateral requirements with financial constraints à la Kiyotaki & Moore (1997) to close the gap between the model and the data, as suggested by Liu, Miao & Zha (2013) or Garin (2015).[15] Liu, Miao & Zha (2013) also incorporate housing-land holdings into the utility function so as to generate countercyclical movements in workers' outside options. However, their model yields limited interactions between labor and financial frictions as labor market variables do not enter the collateral constraint. This can lead them to overestimate the weight of the financial shock, necessary for them to fit the data. Finally, in Petrosky-Nadeau (2013), for the mechanism associated with canonical financial frictions à la Bernanke et al. (1999) to match data, an ad hoc countercyclical monitoring cost is incorporated into the financial contract.[16]

[11]Remark that this source of fluctuation is not shared by Hall (2017).

[12]See table 5, p.1492, of the Krusell et al. (2010) paper and section 5.11.1 for a discussion on the implications of business cycles. Notice that that financial constraints à la Aiyagari (1994) (i.e., exogenous labor market transitions) do not *per se* entail positive welfare effects of eliminating business cycles (see Krusell & Smith (1999)).

[13]See Wasmer & Weil (2004), Petrosky-Nadeau (2013), Petrosky-Nadau & Wasmer (2013), or Zanetti (2015) for an analysis based on real business-cycle models and Zanetti & Mumtaz (2013) and Christiano et al. (2016) for one based on New-Keynesian DSGE models.

[14]Wasmer & Weil (2004), Petrosky-Nadeau (2013), and Petrosky-Nadau & Wasmer (2013) also share this feature.

[15]We also depart from Garin (2015) as we do not introduce fixed training costs and time-varying vacancy, already known to change the prediction of the basic DMP model (even without financial constraints).

[16]Notice that Petrosky-Nadau & Wasmer (2013), Petrosky-Nadeau (2013), Liu, Miao & Zha (2013), and Garin (2015) do not discuss the implications of their models with respect to real wage dynamics (the Keynes-

Figure 2: Getting data-consistent business cycle features and sizeable welfare costs in a search and matching model

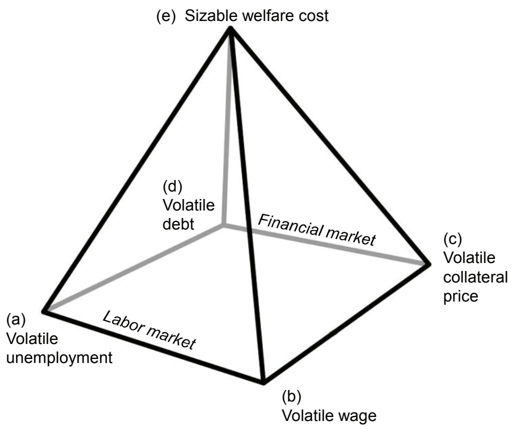


Figure 2 summarizes our contribution to the literature. The standard DMP model with flexible wages, (b) on Figure 2, is in principle associated with large welfare costs of cycles. However, for want of a sufficient unemployment volatility, it fails to generate theses costs. Hairault et al. (2010) introduce rigid wages. This allows them to generate data-consistent volatilities of labor aggregates and entail large welfare costs of cycles – (a) and (e) on Figure 2. However, their model entails counterfactual rigid wages: (b) does not hold. Both frameworks do not consider the interactions between labor and financial markets – and thus, (c) and (d). Petrosky-Nadau & Wasmer (2013) and Petrosky-Nadeau (2013) introduce financial frictions into a DMP model with flexible wages. However, they do not check for debt, collateral nor wages volatility, (b), (c) and (d), nor study welfare implications, (e). Our work make sure that the evaluation of welfare costs, (e), is data consistent on both financial and labor markets, (a), (b), (c) and (d).

3 The model

The economy is populated by two types of agents: firms and workers. The representative firm produces the final consumption good of the economy by combining labor and infrastructure (i.e., land). Firms can finance their activity with loans funded by households. As debt contracts are not complete because of enforcement limits *à la* Kiyotaki & Moore (1997), firms are subject to a collateral constraint. Households include both unemployed individuals and

Tarshis-Dunlop controversy). Moreover, Notice that Garin (2015) does not control for the implications of shocks on financial aggregates. Garin (2015) argues that credit shocks are critical to understanding labor market dynamics. However, according to Figure 3, p.122, of his paper, debt is five times as volatile as output, following a credit shock. This volatility of debt is counterfactual (see Table 1 in this paper). In Liu, Miao & Zha (2013), the estimation includes the real land price (but not corporate debt).

workers employed by the firms. A canonical matching process *à la* Mortensen & Pissarides (1994) allows firms to hire workers. Wages are set according to a standard Nash bargaining process. In order to stress the economic mechanisms at work, we present a streamlined model without capital accumulation: even without capital, households can save by lending to firms. [17] We lay stress on the extensive margin of labor, thereby discarding adjustments in hours, as in Blanchard & Gali (2010).[18] Finally, we consider only technological shocks to allow for a comparison between welfare costs in the model and the literature.

3.1 Labor market flows

The economy is populated by a large number of identical households, normalized to one. Each household consists of a continuum of infinitely living agents. We consider a standard labor and matching model à la citeblanchard-gali-10 and Mortensen & Pissarides (1994). Employment N_t evolves according to

$$N_t = (1 - s)N_{t-1} + M_t \tag{1}$$

where s denotes the separation rate. M_t, the number of hirings per period, is determined by a constant-returns-to-scale matching function $M_t = \chi V_t^{\psi} S_t^{1-\psi}$, with $0 < \psi < 1$, $\chi > 0$ a scale parameter measuring the efficiency of the matching function, and V_t the number of vacancies. Following Blanchard & Gali (2010), we suppose that a pool of jobless individuals, S_t, is available for hire at the beginning of period t. This implies that the pool of jobless agents is larger than the number of unemployed workers. Indeed, individuals are either employed or willing to work (full participation) at all times so that S_t is given by

$$S_t = U_{t-1} + sN_{t-1} = 1 - (1 - s)N_{t-1} \tag{2}$$

where $U_t = 1 - N_t$ is the stock of unemployed workers when the size of the population is normalized to 1 and full participation is assumed. U_t thus measures the fraction of the population left without jobs after hiring takes place in period t. Among agents looking for jobs at the beginning of period t, a certain number M_t is hired, and they start working in the same period. Only workers in the unemployment pool S_t at the beginning of the period can be hired ($M_t \leq S_t$). The ratio of aggregate hires to the unemployment pool is the rate at which jobless people in the pool find a job, $\Psi_t \equiv M_t/S_t$, whereas $\Phi_t \equiv M_t/V_t$ is the rate

[17] We present in section 5.4 an extension with capital.

[18] Following Merz (1995), Langot (1995) and Andolfatto (1996), we assume complete insurance against the unemployment risk. Hence, no wealth heterogeneity exists among workers at the equilibrium.

at which vacancies are filled. Labor market tightness θ_t equals $\frac{V_t}{S_t}$.[19]

3.2 Households

Households maximize the utility function of consumption and labor. In each period, an agent can engage in only one of two activities, working or enjoying leisure. Employment lotteries ensure that the individual idiosyncratic risks faced by agents in their jobs match. Hence, the representative household's preferences are represented by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\mu^t\{N_tU^n\left(C_t^n\right)+(1-N_t)U^u\left(C_t^u+\Gamma\right)\}\right] \tag{3}$$

where $0<\mu<1$ is the discount factor and Γ the utility of leisure. C_t^z stands for the consumption of employed ($z=n$) and unemployed agents ($z=u$). We assume that $U(C_t^n)=\frac{(C_t^n)^{1-\sigma}}{1-\sigma}\equiv\tilde{U}_t^n$ for employed workers and $U(C_t^u+\Gamma)=\frac{(C_t^u+\Gamma)^{1-\sigma}}{1-\sigma}\equiv\tilde{U}_t^u$ for unemployed workers, with $\sigma>0$ the coefficient of relative risk aversion. The budget constraint is

$$[N_tC_t^n+(1-N_t)C_t^u]+B_t\leq R_{t-1}B_{t-1}+N_tw_t+(1-N_t)b_t+T_t \tag{4}$$

where w is the real wage and b the unemployment benefit. T is a lump-sum transfer from the government. Moreover, B represents private bond-financing firms and R is the gross investment return associated with these loans. Households' labor opportunities evolve as follows:

$$N_t=(1-s)N_{t-1}+\Psi_tS_t \tag{5}$$

Each household chooses $\{C_t^n,C_t^u,B_t\}$ to maximize (3) subject to (4) and (5).[20]

3.3 Entrepreneurs and Firms

The economy includes many identical firms. Entrepreneurs maximize the following sum of expected utilities:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^tU\left(C_t^F\right)\right] \tag{6}$$

[19] The jobless rate is thus a convex function of the job-finding rate, $S=\frac{s}{s+(1-s)\Psi}$.

[20] See Appendix B for a complete description of the model.

where β denotes the entrepreneurs' discount factor, and $\mu > \beta$, implying that workers are more patient than firms[21]. Their budget constraint is

$$C_t^F + R_{t-1}B_{t-1} + q_t\left[L_t - L_{t-1}\right] + w_tN_t + \bar{\omega}V_t \leq Y_t + B_t + \pi_t \tag{7}$$

where B is private debt, L productive land or infrastructure, and q its price. Moreover, wN denotes total wages, with N the number of employees, whereas Y is the final output and π lump-sum dividends. Each firm has access to a Cobb-Douglas constant-return-to-scale production technology combining workers and infrastructure (land):

$$Y_t = A_tL_{t-1}^{1-\alpha}N_t^{\alpha} \tag{8}$$

where A_t represents the global productivity of factors in the economy, assumed to evolve stochastically as follows: $\log A_t = \rho_a \log A_{t-1} + (1-\rho_a)\log A + \varepsilon_t^a$, with ε_t^a the iid innovations. Firms's activity can be financed by funds lent by households under imperfect debt contracts. Enforcement limits *à la* Kiyotaki & Moore (1997) imply that entrepreneurs are subject to collateral constraints. As Quadrini (2011) and Jermann & Quadrini (2012), we assume that, at the beginning of the period, firms can access financial markets to finance both their expenditures (i.e., consumption by entrepreneurs and land investments) as well as the costs associated with working capital within the period. Moreover, following Petrosky-Nadeau (2013) and Wasmer & Weil (2004), we suppose for simplicity that the costs associated with working labor represent hiring costs. Entrepreneurs can thus borrow from agents subject to the following collateral constraint:

$$B_t + \omega V_t \leq m\mathbb{E}_t\left[q_{t+1}L_t\right] \tag{9}$$

This constraint shows that if the capitalist fails to repay the loan, the lender can seize the collateral. Given that liquidation is costly, the lender can recover up to a fraction, m, of the value of collateral assets. m is the (exogenous) loan-to-debt ratio. The firms' constraint associated with the evolution of vacancies is

$$N_t = (1-s)N_{t-1} + \Phi_tV_t \tag{10}$$

Each entrepreneur chooses $\{C_t^F, L_t, B_t, V_t, N_t\}$ to maximize (6) subject to (7), (9), and (10),

[21]This assumption is needed to ensure that firms are debt constrained in equilibrium. We will discuss this point in the following.

where Y_t is given by (8). The Job Creation (JC) curve is

$$\frac{(1+\varphi_t)\,\bar{\omega}}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t + (1-s)\,\beta\mathbb{E}_t\left[\frac{\lambda^F_{t+1}}{\lambda^F_t}\frac{(1+\varphi_{t+1})\,\bar{\omega}}{\Phi_{t+1}}\right] \tag{11}$$

where φ_t is the Lagrangian multiplier associated with the credit constraint (Equation (9)) and represents the "credit multiplier" of this model, and λ^F_t is the Lagrangian multiplier associated with the budget constraint (Equation (7)). The credit multiplier (φ) appears both on the LHS and RHS of Equation (11). Nevertheless, there is almost no persistence in the adjustment of φ_t; after a jump at the time of the shock, it comes back to its steady-state value. We thus shift our attention to its impact on the LHS of (11). In recession, tight credit conditions (large values of φ_t) drive up the opportunity costs associated with vacancy posting, $\bar{\omega}\,(1+\varphi_t)$. This introduces a countercyclical and time-varying wedge that has the potential to magnify productivity shocks. If the real wage is sufficiently sluggish, the adjustments of quantities on the labor market can thus be large.

3.4 Wages

The wage is the solution of the maximization of the generalized Nash product $\max_{w_t}\left(\frac{\mathcal{V}^F_t}{\lambda^F_t}\right)^{\epsilon}\left(\frac{\mathcal{V}^H_t}{\lambda_t}\right)^{1-\epsilon}$, with $\mathcal{V}^F_t = \frac{\partial\mathcal{W}(\Omega^F_t)}{\partial N_{t-1}}$ the marginal value of a match for a firm and $\mathcal{V}^H_t = \frac{\partial\mathcal{W}(\Omega^H_t)}{\partial N_{t-1}}$ the marginal household's surplus from an established employment relationship. ϵ denotes the firm's share of a job's value, i.e., firms' bargaining power. The wage curve (WC) is

$$\begin{aligned} w_t \;=\; & \underbrace{\epsilon(b+\Gamma)}_{(a)} + \underbrace{(1-\epsilon)\,\frac{\partial Y_t}{\partial N_t}}_{(b)} \\ & \underbrace{+(1-\epsilon)\,(1-s)\beta\mathbb{E}_t\left[\underbrace{(1+\varphi_{t+1})}_{(3)}\frac{\lambda^F_{t+1}}{\lambda^F_t}\left\{\underbrace{\frac{\bar{\omega}}{\Phi_{t+1}}\frac{\beta\frac{\lambda^F_{t+1}}{\lambda^F_t}-\mu\frac{\lambda_{t+1}}{\lambda_t}}{\beta\frac{\lambda^F_{t+1}}{\lambda^F_t}}}_{(1)} + \underbrace{\frac{\mu\frac{\lambda_{t+1}}{\lambda_t}}{\beta\frac{\lambda^F_{t+1}}{\lambda^F_t}}\bar{\omega}\theta_{t+1}}_{(2)}\right\}\right]}_{(\Sigma)} \end{aligned} \tag{12}$$

where (a) represents the weight of the reservation wage in total wage and $(b)+(\Sigma)$ is the workers' gain from the match. This gain can be decomposed into the marginal productivity of the new employed worker, (b), and the saving on search costs if the job is not destroyed in the next period (Σ).[22] Financial frictions enter the wage equation through term (Σ) and affect wage bargaining directly through the "credit multiplier" φ (term (3)), and indirectly via the gap between agents' discount rates (terms (1) and (2)). If the "credit channel" (term (3)) is also at work in Petrosky-Nadeau (2013) and Wasmer & Weil (2004) and discussed in Monacelli et al. (2011), the impact of the gap between the firm's and the workers' price kernels – i.e., the gap in impatience rates – is a original feature of our model, inducing a gap between agents evaluation of the matching surplus. Hence, the opportunity cost of the time consuming search activity is not measured in the same way by the two parts (Term (1)) and the time duration of the search process is not discounted with the price kernels (term (2)).

3.5 Markets clearing

In order to close the model, we assume that the government does not accumulate debt and pays unemployment benefits using a lump-sum tax; i.e., $T_t = -(1 - N_t)b_t$. It is possible to show (see Becker (1982), Becker & Foias (1982)) that, in presence of standard levels of uncertainty[23], firms are collateral constrained in each period. Thus, the debt limit eventually determines the equilibrium level of corporate debt and workers savings. The private-bond market clears. Good market equilibrium requires the condition $Y_t = N_t C_t^n + (1 - N_t) C_t^u + \bar{\omega} V_t + C_t^F$. Finally, we assume that land supply is fixed and that the land market clears in each period; i.e., $L_t = 1$.

4 Sizeable welfare costs of fluctuations *and* volatile labor-market: a non-trivial coincidence

Nonlinearities arising from the combination of labor market frictions and credit imperfections. Nonlinearities arising from the combination of labor market frictions and credit imperfections are indeed at the roots of our main results, i.e., i) the large volatility of labor-market aggregates in our model and and ii) significant welfare costs associated to fluctuations. In order to stress the role of financial frictions, we first discuss non-linearities intrinsic to the standard DMP model (partial equilibrium analysis). We show how they can *per se* entail both small fluctuations in labor market aggregates (see Shimer (2005)) and

[22] When firms and workers have the same discount factor (let $\mu = \beta$, $\varphi = 0$, thus $\lambda_t = \lambda_t^F$), equation (12) collapses to the standard Blanchard & Gali (2010) wage curve: $w_t = (1-\epsilon)\left(\frac{\partial Y_t}{\partial N_t} + \bar{\omega}(1-s)\beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right]\right) + \epsilon(b + \Gamma)$.

[23] For a discussion see, among others, Iacoviello (2005).

large welfare costs associated to cycles.

In partial equilibrium, welfare costs of fluctuations can be deduced from the gap between the average unemployment (mean of the unemployment in an economy with business cycle fluctuations) and its counterpart in a stabilized economy (deterministic steady state). For simplicity, we focus on conditional steady states, i.e. steady states contingent to each value of y.[24] Welfare costs of fluctuations can thus be proxied as:

$$\begin{aligned}\mathbb{E}(u) - \bar{u} &\approx \frac{s}{[s+\bar{\Psi}]^3}\sigma_\Psi^2 - \frac{s}{[s+\bar{\Psi}]^2}\left(\mathbb{E}(\Psi) - \bar{\Psi}\right) \\ &\approx \underbrace{\frac{s}{[s+\bar{\Psi}]^3}\sigma_\Psi^2}_{\substack{\text{"Beveridge"} \\ \uparrow \text{ welfare costs}}} - \frac{s}{[s+\bar{\Psi}]^2}\frac{1}{2}\underbrace{\left(\underbrace{\Psi''(\bar{\theta})(\theta'(\bar{y}))^2}_{\substack{\text{"JFR"} \\ \uparrow \text{ welfare costs}}} + \underbrace{\Psi'(\bar{\theta})\theta''(\bar{y})}_{\substack{\text{"Hiring policy"} \\ \downarrow \text{ welfare costs}}}\right)}_{=G}\sigma_y^2 \qquad (13)\end{aligned}$$

Equation (13) shows that the gap between $\mathbb{E}(u)$ and $\bar{u}$ depends on 3 elements:

(i.) the term labeled "Beveridge". Labor market flows imply a convex relationship between the job finding rate and the unemployment rate. Hence, fluctuations are *per se* costly. Indeed, the mean of unemployment in an economy that fluctuates is greater than its counterpart in a stabilized economy. [25]

The gap between the average job finding rate Ψ and its steady-state value consists in turn of 2 terms.

(ii.) the term labeled "JFR" refers to the concavity of the job finding rate with respect to market tightness, ($\Psi''(\theta) < 0$). The concave relation is implied by the matching function. In a recession, when many workers are unemployed and the job finding is rate low, a small change in labor market tightness (through more vacancy openings for instance) significantly affects the job finding probability. In a boom, the job finding rate is high. A higher value of θ increases the job finding rate, but less than it decreases

[24]For the sake of clarity, we also assume temporarily that the economy jumps from a steady state to another without transition and we use the Taylor development $\mathbb{E}[\Psi] \approx \mathbb{E}[\overline{\Psi} + \Psi'\theta'(y-\overline{y}) + (1/2)(\Psi''(\theta')^2 + \Psi'\theta'')(y-\overline{y})^2]$.

[25]Suppose that the job finding rate follows a Markov process, then steady state unemployment is $\frac{s}{s+\sum_i \pi_i \Psi_i} = \frac{s}{s+\mathbb{E}(\Psi)}$, which is lower than average unemployment given by $\sum_i \pi_i \left(\frac{s}{s+\Psi_i}\right) \approx \mathbb{E}(u)$ where π_i is the occurrence of the states i. In this section, for the purpose of brevity, we assume that the variance of the job finding rate (σ_Ψ^2) is given and is at its observed level. In section 5, we will show how financial frictions play a significant role in helping the model replicate the volatility of the job finding rate at the general equilibrium.

it during recessions. According to Equation (13), the term "JFR" tends to increase welfare costs of fluctuations.

(iii.) the term labeled "Hiring policy" refers to the convexity of labor market tightness with respect to productivity, ($\theta''(y) > 0$). As the matching function exhibits decreasing marginal returns to vacancies, the free entry condition is satisfied for greater variations in job creation during booms than during recessions. According to Equation (13), the term "Hiring policy" tends to reduce welfare costs of fluctuations.

Hence, without financial frictions, these two last forces act in opposite directions in the standard DMP model: *(ii.)* the concavity of the matching function leads the average job finding rate to be lower than its stabilized counterpart – inducing greater costs of fluctuations–, whereas *(iii.)* the free entry condition prompts firms to react more in booms than in recessions so as to compensate concavity's implications of hiring costs – increasing employment and thus improving welfare. The net effect is summarized by term G in Equation (13). The more negative G, the greater the welfare costs of fluctuations.

We now investigate the impact of financial frictions on term G, in presence of flexible wages. We first compute the equilibrium labor market tightness, $\theta(y)$ (i.e., the intersection of the job creation curve and the wage curve). With financial frictions, $\theta(y)$ is given by:

$$\frac{\bar{\omega}\left(1+\varphi(y)\right)}{\Phi(y)} = \frac{y - w(\theta(y),y)}{1-(1-s)\,\beta} \tag{14}$$

$$\text{where} \quad \begin{cases} w(\theta(y),y) &= \epsilon(b+\Gamma) + (1-\epsilon)\left(y + \Sigma(\theta(y),y)\right) \\ \Sigma(\theta(y),y) &= (1-s)\frac{\beta}{1-\varphi(y)}\left(-\frac{\bar{\omega}(1+\varphi(y))}{\Phi(\theta(y))}\varphi(y) + \bar{\omega}(1+\varphi(y))\theta(y)\right) \\ \Phi(\theta(y)) &= \chi\theta(y)^{\psi-1} \\ \varphi(y) &= 1 - \frac{\beta}{\mu}y^{\zeta} \end{cases}$$

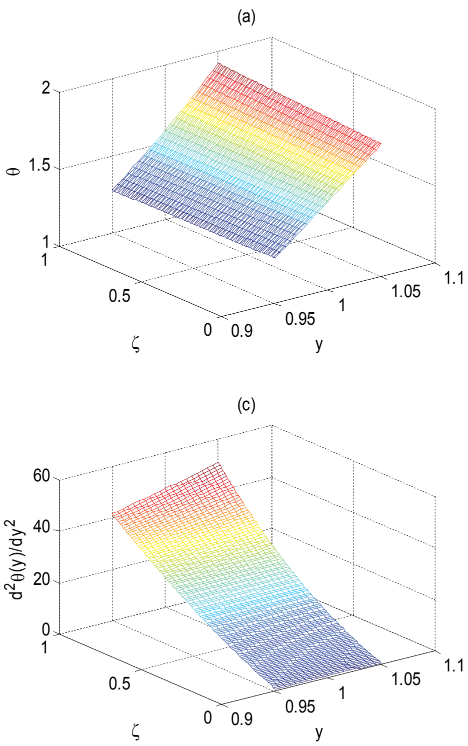
with $\mathbb{E}(y) = \overline{y} = 1$. We approximate the impact of financial frictions on the labor market by term $\varphi(y)$ where $\zeta > 0$. Consistently with our model, this entails $\varphi'(y) < 0$ (i.e., during booms, the cost of credit decreases).[26] When $\zeta = 0$, there are no financial frictions and we recover the standard DMP model with flexible wages. We can now calculate the gap between $\mathbb{E}(\Psi)$ and $\bar{\Psi}$ in presence of financial frictions. To this purpose, we compute $\theta'(y)$ and $\theta''(y)$ and finally G. These functions are displayed in Figure 3.[27]

The first interesting result comes from the shape of $\theta'(y) = \frac{d\theta(y)}{dy}$ (panel (b) of Figure 3).

[26] We also assume that the upper bound of the y-distribution is lower than $(\mu/\beta)^{1/\zeta}$, so that $\varphi(y) > 0$.

[27] Not surprisingly, the vacancy-unemployment ratio is an increasing function of y (panel (a) of Figure 3). Moreover, the weight of financial frictions, governed by the parameter ζ does not change so much the level of $\theta(y)$.

Figure 3: The functions $\theta(y)$, $\theta'(y)$, $\theta''(y)$ and G with or without financial frictions. Flexible wages



Calibration: $\omega = 0.17$, $\chi = 0.5$, $\epsilon = 0.5$, $b = 0.4$, $\Gamma = 0.3$, $s = 0.05$, $\beta = .998$, $\beta/\mu = 0.99$.

Notice that the case of the standard DMP model (i.e.,$\zeta = 0$) predicts the lowest responsiveness of labor market tightness to productivity. This entails low volatility of labor-market aggregates (consistently with Shimer (2005)). However, as financial frictions increase (ζ goes up), the sensitivity of $\theta(y)$ to changes in the exogenous process y is magnified. Higher responsiveness of the credit costs to productivity (associated to greater values of ζ) helps thus the model predict more volatile labor market tightness, which makes the model consistent with data. In our quantitative analysis (section 5) we will show the implications of these mechanisms for the business cycle properties of our model.

Panel (c) of Figure 3 reports the convexity of labor market tightness with respect to productivity, ($\theta''(y) = \frac{d^2\theta(y)}{dy^2} > 0$). Without financial frictions ($\zeta = 0$), this convexity term is small. Therefore, the "JFR" term dominates the "Hiring policy" term. The standard DMP model thus predicts a very negative G (Panel (d) of Figure 3), thereby generating large gaps between $\mathbb{E}(u)$ and $\bar{u}$. In a nutshell, the DMP model is thus potentially associated with significant costs of fluctuations. However, it is also characterized by a very low responsiveness of labor market tightness to productivity (which is not consistent with data). In absence of volatile labor market tightness, the strong non-linearities are actually unable to generate sizable welfare costs of the business cycle.

When financial frictions distort the equilibrium ($\zeta > 0$), function $\theta(y)$ becomes highly convex (i.e., $\theta''(y) = \frac{d^2\theta(y)}{dy^2}$ increases) and labor aggregates are very sensitive to productivity shocks. Indeed, in booms, lower credit costs reduce hiring costs; this prompts in turn an increase in job creation. In parallel, as the the term "hiring policy" becomes very convex (when ζ increases), the "JFR" component can actually be dominated by the "hiring policy" term. Notice that when ζ is large, term G (Panel (d) of Figure 3) can even be positive, thereby reducing the gap between $\mathbb{E}(u)$ and $\bar{u}$ (Equation (13)) – and thus the cost of cycles. Therefore, while entailing greater responsiveness of the labor market to productivity, financial frictions are not systematically associated with greater welfare costs. Indeed, when credit cost is very responsive to the productivity shock, firms benefit so much from the falling recruitment costs in expansion that the average job finding rate lies above its steady state value, thereby reducing welfare costs of fluctuations.

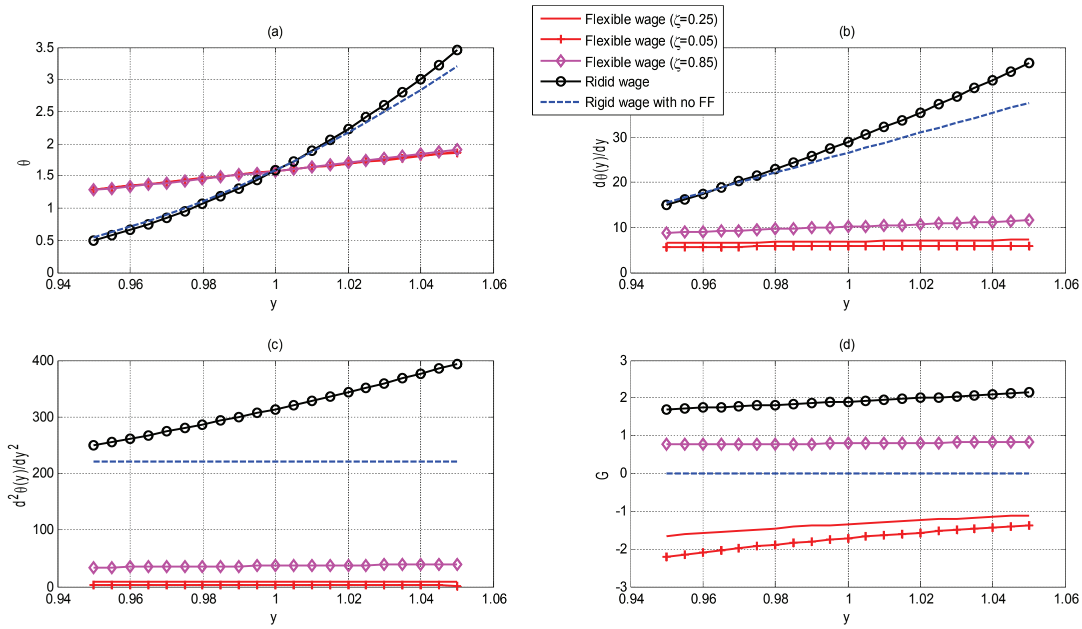
The role of wage bargaining. The above analysis has investigated how the standard DMP model is structurally characterized by significant welfare costs of cycles but cannot reproduce the strong responsiveness of labor aggregates to productivity. Hairault et al. (2010) introduce rigid real wages à la Hall (2005)[28] (see Figure 4, panel (b), "rigid wage and No FF") and compute the welfare cost of business cycles. However, with rigid wages, their model predicts a counter-factual zero wage volatility, while in US data wages do respond to the business cycle (wage volatility in US data is 60% of that of output).[29]

Figure 4 displays the components of G under different scenarios of wages (rigid vs. bargained) and for increasing values of ζ. When wages are rigid, with financial frictions, the financial accelerator makes the labor market very responsive to productivity shocks, together with labor-market tightness (panel (b) on Figure 4). However, at the same time, job creation becomes very convex (panel (c) on Figure 4) so that only fluctuations in the hiring costs matter in job-creation decisions. During booms, as credit conditions improve, hiring costs fall drastically. Because of rigid wages, firms are not constrained by wage increases and

[28]Remark that a rigid wage can be viewed as an extreme case of a calibration à la Hagendorn & Manovskii (2008). This interpretation is neutral for the business cycle properties of the model, but not for the welfare implications because the value of unemployment is close to the firm productivity, and this leads to questionable negligible welfare costs of unemployment.

[29]Hairault et al. (2010) also kill the gap between $\mathbb{E}(\Psi)$ and $\bar{\Psi}$ induced by the business cycle. In Equation (3), $G = 0$ (panel (d) of Figure 4) the "JFR" term exactly compensates for the "Hiring policy" term in Equation (3) and thus only the "Beveridge" term matters for the cost of the business cycle, as in a partial equilibrium analysis.

Figure 4: Why does wage flexibility matter?



Common parameters: $\epsilon = 0.5$, $b = 0.4$, $\Gamma = 0.3$, $s = 0.05$, $\beta = .998$, $\beta/\mu = 0.99$.
Specific parameters :
-Flexible wage: $\omega = 0.17$, $\chi = 0.5$.
-Flexible wage ($\zeta = 0.05$): standard DMP model
-Rigid wage: $\omega = 0.18$, $\chi = 0.1$, implying $\theta(y=1)|_{FW} = \theta(y=1)|_{RW}$ for $\overline{w} = 0.88$. $\zeta = 0.25$
-Rigid wage & no FF (Hairault et al. (2010)): $\omega = 0.183$, $\chi = 0.1$, implying $\theta(y=1)|_{FW} = \theta(y=1)|_{RW}$ for $\overline{w} = 0.88$.

vacancy posting jumps up. This effect is so strong that it tends to dominate, so that G becomes positive (panel (d) in Figure 4). This dampens welfare costs and can potentially entail welfare gains of fluctuations. Therefore, with financial frictions, wage rigidity actually *reduces* welfare costs of fluctuations.

In presence of financial frictions, a bargained wage is a key ingredient to generate sizeable welfare costs. The firm shares the financial cost of credit with workers. Financial imperfections affect the wage equation throughout the cycle because of 2 effects: (1) in expansion, due to congestion effects, it takes longer for the firm to fill a vacancy, so that the firm borrows for a longer time, increasing its hiring costs; (2) during booms, the unit cost of this credit is low, thereby reducing hiring costs. (1) makes the firm reluctant to accept higher wages in booms. (1) is a countercyclical component of wages, which helps the model predict a high volatility of labor market tightness, while (2) makes wages higher in booms, which reduces the firm's incentive to open more vacancies. Panel (b) in Figure 4 shows that (1) dominates (2). Indeed, under flexible wages, $\frac{d\theta(y)}{dy}$ increases with ζ. Moreover, $\theta'(y)$ remains below the values predicted under rigid wages. This happens because, in presence of financial frictions, the wage still responds procyclically to productivity shocks and this tends to dampen the response of labor market tightness to productivity. Interestingly, the presence of flexible wages puts a lid on the convexity of the hiring policy (panel (c) Figure 4). Notice that, even if wages increase (effect that is dampened by the increase in the credit cost), firms still open more vacancies in expansion than in recession, but less than in the case of rigid wages and more than without financial frictions. Finally, panel (d) in Figure 4 recalls that financial frictions and wage flexibility do not systematically produce large welfare costs of fluctuations (G can actually be positive if ζ is large).

In summary, introducing financial frictions into the standard DMP model with Nash-bargained wages can increase labor market fluctuations (with data-consistent wage volatility) *and* generate sizeable welfare cost of cycles. This is not a trivial outcome.[30]

5 Quantitative analysis

In this section, we present the calibration and show that our model is able to match the magnitude of cycles. Indeed, for our quantitative calculation of business-cycle costs to be relevant, we need to match the volatility of data. Overcoming the "Shimer puzzle" is thus a necessary condition for our exercise. We then measure the welfare costs of business cycles.

5.1 Calibration

The calibration is based on quarterly US data. Data are described in Appendix A. Table 4 in Appendix A shows all parameters and targets.

Preference, technology and shocks. The discount factor for patient agents is consistent with a 4% annual real interest rate. For the impatient consumer, we set $\beta = 0.99$, which is within the range of values chosen by Iacoviello (2005)[31]. The risk aversion is set to 1 for firms and 2 for workers. Both values lie within a standard interval in the literature. In addition, the firm is characterized by a lower risk aversion because, as shown by Iacoviello (2005), such a calibration ensures that the borrowing constraint is binding for a wide range of volatility shocks, impatience levels, and loan-to-value ratios (m values). The technological shock is calibrated as in Hairault et al. (2010). We choose the standard deviation of technological shock to reproduce the observed GDP standard deviation.

[30] In what follows, we will check that our benchmark calibration verifies parameter restrictions at the general equilibrium so as to obtain these predictions.

[31] Notice that an extremely low degree of impatience heterogeneity is sufficient for debt limits to hold.

Financial frictions. The corporate debt-to-GDP ratio pins down the value of m in the collateral constraint. To this end, we use the average corporate debt over GDP for 2001-2009 (debt outstanding, annual data, corporate sector, Flow of Funds Accounts tables of the Federal Reserve Board).

Labor market. Employment level N is consistent with the average unemployment rate ($N = 0.88$) estimates of Hall (2005).[32] As in Shimer (2005), the quarterly separation rate s is 0.10, so jobs last for about 2.5 years on average. Using steady-state labor-market flows, we infer Ψ given s and N. This leads to $\Psi = 0.423$. This value is lower than in the usual DMP model. Indeed, the pool of job seekers is larger in Blanchard & Gali (2010) than in the standard DMP model. The elasticity of the matching function with respect to the number of job seekers is $\psi = 0.5$, which lies within the range estimated in Petrongolo & Pissarides (2001). The efficiency of matching, χ, is set such that firms with a vacancy find a worker within a quarter with a 95% probability, which is consistent with Andolfatto (1996). The cost of posting a vacancy, ω, is set to 0.17 as in Barron & Bishop (1985) and Barron et al. (1997). We obtain $\frac{\omega V}{Y} = 0.0179$, which is within the range found in the literature (0.005 in Chéron & Langot (2004) or 0.05 in Krause & Lubik (2007)). The utility of leisure parameter, Γ, is pinned down so as to match an income of the unemployed equal to 0.6.[33] We obtain $\Gamma = 0.29$, leading to $b + \Gamma = 0.89$, a lower value than one used by Hagendorn & Manovskii (2008) ($b = 0.95$ with $\Gamma = 0$). It is also lower than the parameters combination allowing Hall & Milgrom (2008) to control for the unemployment rate, namely the sum of leisure value ($b = 0.71$) and the "employer's cost of delay" ($\gamma = 0.27$) which leads to $b + \gamma = 0.98$.[34]

5.2 Business-cycle properties

In this section, we document the unconditional business cycles facts on financial variables and labor market adjustments. Our contribution lies also in bringing together financial data (from Jermann & Quadrini (2012)) and data from the labor market literature (Shimer

[32]According to Hall (2005), the observed high transition rate from "out of the labor force" directly to employment suggests that a fraction of those classified as out of the labor force are nonetheless effectively job-seekers. Hall (2005) adjusts the US unemployment rate to include individuals out of the labor force who are actually looking for a job.

[33]The replacement ratio (in the initial phase of unemployment) reported for the US by the OECD is equal to 62% for a single earner with a previous wage equal to 2/3 of the average wage. To go beyond than the study of typical cases, Mulligan (2012) estimates that the median replacement rate is equal to 63% in the US, if the variety of income support programs is taken into account.

[34]As in Hall & Milgrom (2008), we will show that this is not the levels of these parameters that imply the labor market volatilities, but, in our case, the financial friction in interaction with the wage bargaining, whereas in Hall & Milgrom (2008) it comes from the frequency of bargaining interruptions.

(2012)). In both markets, we focus on fluctuations in quantities (debt, unemployment) as well as equilibrium prices (interest rate, wage).[35] Table 1, column 1, reports business-cycle properties found in the data.

Table 1: Business-cycle volatility : Models versus data

	(1) Data std(.)		(2) Benchmark Model std(.)		(3) Without Financial Frictions std(.)	
Y	1.44	**	1.44	**	1.44	**
C	0.81	*	0.88	*	0.94	*
N	0.72	*	0.74	*	0.46	*
Y/N	0.54	*	0.28	*	0.56	*
w	0.62	*	0.49	*	0.49	*
U	7.90	*	5.45	*	3.71	*
Ψ	5.46	*	6.26	*	2.79	*
V	9.96	*	12.7	*	4.60	*
B	1.68	*	1.35	*		
q	3.21	*	2.59	*		
R	0.92	*	0.32	*		
$corr(U, \Psi)$	-0.91		-0.86		-0.92	
$corr(U, V)$	-0.97		-0.71		-0.76	

** std (in percentage); * relative to GDP std
$\sigma_A = 0.0031$ in column (2); $\sigma_A = 0.0063$ in column (3)

The volatility of real wages is not close to zero. Moreover, it is larger than that of labor productivity. This clearly suggests that real wage rigidity (implying a zero standard deviation for fluctuations in w) is not a realistic explanation for the strong cyclicality of labor market aggregates. $corr(U_t, V_t)$ summarizes the dynamics around the Beveridge curve. The negative covariance is consistent with the view that aggregate shocks have a more important weight than reallocation shocks at business-cycle frequency. As expected, $corr(U_t, \Psi_t)$ is negative. Jung & Kuester (2011) point out that mean unemployment exceeds steady-state unemployment when the job-finding rate and the unemployment rate are non-positively correlated and the average job-finding rate is lower than the steady-state job-finding rate.[36] This is

[35] All data have been recomputed and updated so that our sample covers five recession episodes from 1976 January through January 2013 (see Appendix A for a complete description of the data set). Previous works that study the interaction between financial and real variables in DSGE models such as Monacelli et al. (2011) and Christiano et al. (2010) summarize labor market adjustments using only fluctuations in employment and unemployment.

[36] This can be inferred from the employment-flow equation taken at the steady state $sN_t = \Psi U_t$ where $N_t = 1 - U_t$. Hence, $s\mathbb{E}(1_t - U_t) = cov(U_t, \Psi_t) + \mathbb{E}(U_t)\mathbb{E}(\Psi_t)$. Subtracting the steady-state from both sides of the above equation, leads to $\mathbb{E}(U_t) - u_t = -\frac{1}{s+\Psi}\left[cov(U_t, \Psi_t) + (\mathbb{E}(\Psi_t) - \Psi_t)\,\mathbb{E}(U_t)\right]$. We deduce that if (i) $\mathbb{E}(\Psi_t) - \Psi_t < 0$ and (ii) $cov(U_t, \Psi_t) < 0$, then necessarily, $\mathbb{E}(U_t) - u_t > 0$. The correlation at the bottom of

the case in our model. Comparing columns 1 and 2 of Table 1, we note that the model generates volatile employment, vacancies, unemployment, and job-finding rates. The simulated volatilities are even a bit higher than the observed volatilities, as we tend to slightly underestimate wage volatility. Notably, the predicted volatile adjustments on the labor market are not obtained under unrealistic fluctuations on financial markets. The model also reproduces fluctuations of corporate debt, nearly as volatile as in the data. Same considerations apply to the dynamics of land prices q. When the model is simulated without financial frictions (Table 1, column 3) and with a TFP process adjusted to match the volatility of output[37], the standard deviation of the job-finding rate relative to GDP is twice lower. Moreover, the relative standard deviation of the wage is the same as in the model with financial frictions, whereas the volatility of the outside option (given by $\widehat{\theta} = \frac{1}{\psi}\widehat{\Psi} > \widehat{\Psi}$) is more than twice as small. This clearly confirms that financial frictions dampen large movements in workers' outside options in the Nash bargained wage rule.

Matching volatility on the labor market: main mechanisms at work. To understand the mechanisms at work, we log-linearize both (JC) and (WC) curves. For simplicity, we assume that φ_t is an uncorrelated process, implying $\mathbb{E}_t[\widehat{\varphi}_{t+1}] = 0$.[38] The log-linearized (JC) is

$$\frac{\varphi}{1+\varphi}\widehat{\varphi}_t + (1-\psi)\widehat{\theta}_t = \frac{\Phi y}{\bar{\omega}(1+\varphi)}\widehat{y}_t - \frac{\Phi w}{\bar{\omega}(1+\varphi)}\widehat{w}_t + (1-s)\,\beta\mathbb{E}_t\left[(1-\psi)\widehat{\theta}_{t+1}\right]$$

Notice that, for a given level of wages ($\widehat{w}_t = 0$), countercyclical movements in φ_t amplify the response of labor-market tightness θ_t. This first mechanism is due to the fact that the expected cost to fill a vacancy is augmented by an extra (credit) cost. Notice however that here, differently from Pissarides (2009), this additional hiring cost (the financial cost) is countercyclical.[39] Moreover, when wages are Nash-bargained, (i.e., $\widehat{w} \neq 0$) they are procyclical. Higher wages tend to absorb the increase in productivity, thereby dampening the hiring incentive – and hence the responsiveness of $\widehat{\theta}_t$. Therefore, for labor-market tightness to be strongly responsive, the wage must incorporate a component that counterbalances the

Table 1 suggests that *ii)* holds in the data.

[37]Tho obtain this level of volatility the standard deviation of the technological shock must be multiplied by two.

[38]This is not the case at the general equilibrium, but the IRF of the model suggest that it is a acceptable simplifying assumption.

[39]This mechanism is also present in Petrosky-Nadeau (2013).

increase in workers' outside options during booms. The log-linearized (WC) curve is[40]

$$\widehat{w}_t = \frac{(1-\epsilon)y}{w}\widehat{y}_t + \frac{(1-\epsilon)\Sigma}{w}\widehat{\Sigma}_t \qquad \text{with} \tag{15}$$

$$\widehat{\Sigma}_t = \underbrace{\frac{\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\mathbb{E}_t[\widehat{\theta}_{t+1}]}_{\text{workers' outside options}} \underbrace{\underbrace{-\frac{\frac{\bar{\omega}}{\Phi}(\beta-\mu)}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\mathbb{E}_t[\widehat{\Phi}_{t+1}]}_{(1)} \underbrace{-\frac{\beta-\mu}{\beta}\frac{\frac{-\bar{\omega}}{\Phi}\mu+\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\widehat{\varphi}_t}_{(2)}}_{\text{Credit cost}} \tag{16}$$

Financial frictions change the dynamics of workers' outside options ($\widehat{\Sigma}_t$) with respect to the DMP model (see equation (16)).[41] The credit-cost term has two components: (1) the duration of the borrowing period, which is given by the job-filling rate ($\widehat{\Phi}_t$) and (2) the tightness of the collateral constraint ($\widehat{\varphi}_t$). Because it is associated to the length of the borrowing period, term (1) provides a measure of the quantity of credit; term (2) provides a measure of the unit cost of credit.

(1) *Fluctuations in the time duration to fill a vacancy: the quantity of credit.* In expansion, the average time to fill a vacancy increases[42] and thus time during which the firm borrows. Hence, the quantity of credit the firm needs to borrow increases in booms. This component is a countercyclical force in the wage equation if $-\frac{\frac{\bar{\omega}}{\Phi}(\beta-\mu)}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta} > 0$, which is always satisfied for a β value sufficiently close to μ.

(2) *Fluctuations of the impatience gap: the unit cost of credit.* These fluctuations are given by $\widehat{\varphi}_t$, which is countercyclical.[43] In booms, the gap between workers' and firms' discount factors falls. Hence, the unit credit cost is reduced ($\widehat{\varphi}_t < 0$).[44] The total impact of the gap between workers' and firms' stochastic discount factors (the unit credit cost) is unambiguously pro-cyclical. Indeed, $-\frac{\beta-\mu}{\beta}\frac{\frac{-\bar{\omega}}{\Phi}\mu+\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta} < 0$, for β sufficiently close to μ.

[40] Recall that we assume that φ_t is an uncorrelated process, implying $\mathbb{E}_t[\widehat{\varphi}_{t+1}] = 0$.

[41] Together with the associated stochastic multipliers.

[42] The probability of filling a vacancy is such that $\widehat{\Phi}_{t+1} = (\psi-1)\widehat{\theta} < 0$.

[43] We use the following approximation of the Euler equations on consumption:

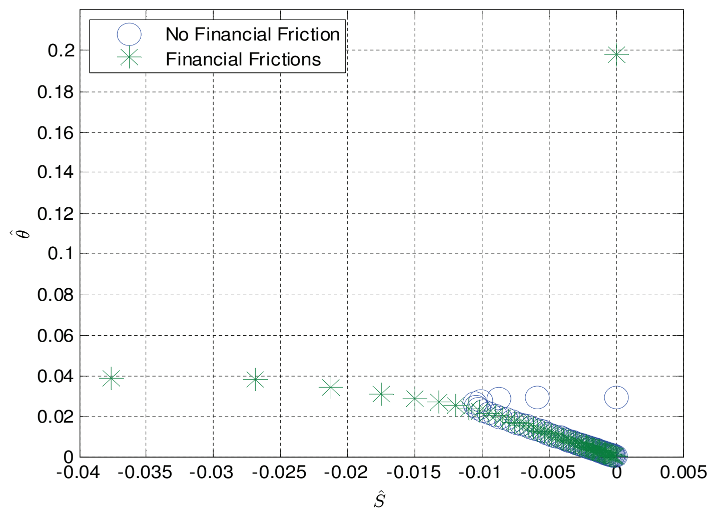
$$\left.\begin{array}{rcl} R_t\mu\mathbb{E}_t\left[\lambda_{t+1}\right] &=& \lambda_t \\ R_t\beta\mathbb{E}_t\left[\lambda^F_{t+1}\right] &=& (1-\varphi_t)\lambda^F_t \end{array}\right\} \approx \left\{\begin{array}{rcl} \widehat{R}_t + \mathbb{E}_t\left[\widehat{\lambda}_{t+1}\right] &=& \widehat{\lambda}_t \\ \widehat{R}_t + \mathbb{E}_t\left[\widehat{\lambda}^F_{t+1}\right] &=& -\frac{\varphi}{1-\varphi}\widehat{\varphi}_t + \widehat{\lambda}^F_t \end{array}\right.$$

$$\Rightarrow \mathbb{E}_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \mathbb{E}_t\widehat{\lambda}^F_{t+1} + \widehat{\lambda}^F_t = \frac{\varphi}{1-\varphi}\widehat{\varphi}_t$$

[44] Remark that the same force acts in the opposite direction during the bargaining process, but it is always offset. Indeed, entrepreneurs are more patient during expansions, leading them to delay negotiations so as to pay lower wages.

These two components affect wages in opposite directions: in booms, as firms wait longer before recruiting a worker, the quantity of credit expands, while the unit cost of credit falls. Notice that effect (1) actually dominates effect (2). In appendix B.4, we study why it is the case in general equilibrium. We also study the restrictions that let financial frictions make labor-market aggregates more responsive (and wages more sluggish). This endogenous wage rigidity leads to large adjustment in labor market quantities (short run overshooting). Figure 5 shows the impulse response function θ and joblessness S in response to a productivity shock, in the models both with and without financial frictions (benchmark calibration at the general equilibrium). In response to the shock, financial frictions trigger a strong instantaneous reaction of market tightness (and thus unemployment). This confirms that the volatility of labor market aggregates is amplified by financial frictions.

Figure 5: Financial frictions amplify the response of labor market aggregates



Impulse Response Function (IRF) in the model with and without financial frictions. Both IRFs start at 0, the period of the shock. Benchmark calibration. hat-variable denotes deviation from the steady state.

Financial fictions thus introduce an important countercyclical component into wage dynamics. This component is absent in Pissarides (2009) and it is much stronger than in Petrosky-Nadeau (2013). Indeed, they do improve the bargaining position of the firms – by reducing the search costs (see component $\omega(1+\varphi(y))\theta(y)$ in Equation (14)), as in Petrosky-Nadeau (2013). But they also reduce the search value demanded by workers during the bargaining process (i.e., component $\frac{\omega(1+\varphi(y))}{\Phi(\theta(y))}\varphi(y)$ in the Equation (14)). This latter countercyclical term is specific to financial frictions à la Kiyotaki & Moore (1997) and is due to the heterogeneity in discount rates. Indeed, given that i) the quantity of credit firms need to borrow increases in booms and that ii) entrepreneurs are impatient ($\beta < \mu$), the first periods of search weight more for the latter. This prompts entrepreneurs to discount the match surplus less and become reluctant to pay higher wages.

Matching volatility on the financial market : main mechanisms at work. The model is able to replicate the volatility on the debt market. First, as the quantity of land does not adjust over the business cycle (which is a realistic assumption), any change in land demand results in a change in the real estate price q. Our model is then able to match the volatility of real-estate prices q.

As for the volatility of corporate debt B, it is noticeable that the model also reproduces a realistic volatility. Indeed, let us consider a log-linearized version of the binding borrowing constraint. Using a log-linearized version of Equation (9), the variance of the price of collateral appears to equal the sum of the variance of corporate debt, vacancies and the covariance between debt and vacancies. In a model without interaction between labor and financial frictions, vacancies would not appear in the borrowing constraint, so that the variance of the price of collateral would equal the variance of corporate debt. In that case, the model would predict fluctuations in corporate debt as large as real-estate price q, which is counterfactual. In contrast, in our model, with the presence of vacancies in the borrowing constraint, the model can predict a large variance of real-estate price as well as a lower variance of corporate debt because of the impact of the large variance of vacancies and the positive covariance between debt and vacancies. Hence, the interaction between labor and financial frictions also make the model consistent with the observed volatility of financial variables.

5.3 Welfare cost of fluctuations

To measure implied business-cycle costs, we compute simulated paths using a second-order approximation of decision rules. This allows us to take into account nonlinearities and business-cycle asymmetries.

Decomposing the welfare cost of fluctuations. The expected lifetime utility of a worker is $\tilde{U}^w = \mathbb{E}_0 \sum_{t=0}^{\infty} \mu^t U\left(C_t + (1 - N_t)\Gamma\right)$, because $C_t \equiv N_t C_t^n + (1 - N_t) C_t^u$ and the FOC on consumption imply $C_t^n = C_t^u + \Gamma$. We define the welfare costs associated with business cycles, τ, as the fraction of steady-state consumption that workers would give up to be indifferent between the steady state and the fluctuating economy. The welfare cost of fluctuations τ is such that $\sum_{t=0}^{\infty} \mu^t U\left([\bar{C} + (1 - \bar{N})\Gamma](1 - \tau)\right) = \tilde{U}^w$, where variables marked with an overbar denote their steady-state values.[45] We deduce

$$\tau = 1 - \left[\tilde{U}^w \frac{(1-\mu)(1-\sigma)}{(\bar{C}+(1-\bar{N})\Gamma)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$$

The result is reported in Table 2, line 1, column A. The business-cycle cost of fluctuations with financial frictions is 2.50% of workers' permanent consumption. This number is far larger than the estimates found by Lucas (1987, 2003), who reports a welfare cost of $\tau = 0.05\%$ with log utility. Notably, welfare costs are large even though agents can save by lending to firms; workers can actually smooth business cycles with savings.

One way to understand these quantitative results is to decompose the welfare cost into a "level effect" and a "business-cycle effect". The former component is due to the gap between the steady-state level of consumption and its counterpart in the stabilized economy. The latter, the "business-cycle effect", entails costs that are directly associated to the volatility of the economy, as in Lucas (1987, 2003). We use a Taylor expansion of welfare in the volatile economy. The crucial point at this stage is to consider the Taylor expansion around the mean of the stochastic economy, and not around the deterministic steady state. This ensures that the computation takes into account the gap between the mean (i.e., the stochastic steady state) and the deterministic steady state. We approximate welfare in the volatile economy as the sum of a consumption gap and Lucas' original measure. Indeed, we have

$$\tilde{U}^w \approx \frac{1}{1-\mu}U\left(\mathbb{E}_0[C+(1-N)\Gamma]\right)\left[1-\frac{1}{2}\sigma(1-\sigma)\left(\gamma_c Var(\widehat{c})+\gamma_u Var(\widehat{u})+\gamma_{cu}Cov(\widehat{c},\widehat{u})\right)\right]$$

where we denote[46] $\widehat{x} = \frac{X_t-\mathbb{E}_0[X]}{\mathbb{E}_0[X]}$, for $x = C, U$ and $\gamma_c = \frac{\mathbb{E}_0[C^2]}{\mathbb{E}_0[(C+(1-N)\Gamma)^2]}$, $\gamma_u = \frac{\Gamma^2\mathbb{E}_0[(1-N)^2]}{\mathbb{E}_0[(C+(1-N)\Gamma)^2]}$ and $\gamma_{cu} = \frac{2\Gamma\mathbb{E}_0[C(1-N)]}{\mathbb{E}_0[(C+(1-N)\Gamma)^2]}$. This leads to

$$1-\tau \approx \underbrace{\left(\frac{\mathbb{E}_0[C+(1-N)\Gamma]}{\bar{C}+(1-\bar{N})\Gamma}\right)}_{=1-\tau_{Gap}}\underbrace{\left[1-\frac{1}{2}\sigma(1-\sigma)\left(\gamma_c Var(\widehat{c})+\gamma_u Var(\widehat{u})+\gamma_{cu}Cov(\widehat{c},\widehat{u})\right)\right]^{\frac{1}{1-\sigma}}}_{=1-\tau_{BC}}$$

If we neglect the level effect, then we have $U\left(\mathbb{E}_0[C+(1-N)\Gamma]\right) \approx U\left(\bar{C}+(1-\bar{N})\Gamma\right)$ because we assume that $\mathbb{E}_0[C+(1-N)\Gamma] \approx \bar{C}+(1-\bar{N})\Gamma$, then $\tau = \tau_{BC}$ where τ_{BC} denotes

[45] A similar computation is possible for the firm's owner: $\sum_{t=0}^{\infty}\beta^t U\left(\bar{C}^F(1-\tau^F)\right) = \tilde{U}^F = \sum_{t=0}^{\infty}\beta^t U\left(C_t^F\right)$. When introducing financial frictions, firms' welfare costs of fluctuations can also be taken into account. In this case, aggregate welfare costs would then be greater. We choose to focus on workers' welfare costs only to compare our results to the existing literature.

[46] Indeed, given that $\frac{\partial U}{\partial C} = U'$, $\frac{\partial U}{\partial N} = -\Gamma U'$, $\frac{\partial^2 U}{\partial C^2} = U''$, $\frac{\partial^2 U}{\partial N^2} = \Gamma^2 U''$ and $\frac{\partial^2 U}{\partial C \partial N} = -\Gamma U''$, we obtain, with the usual functional form $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$, $U' = x^{-\sigma}$ and $U'' = -\sigma x^{-\sigma-1} = -\sigma(1-\sigma)\frac{x^{1-\sigma}}{1-\sigma}\frac{1}{x^2}$.

the welfare costs of the business cycle computed in the spirit of Lucas (1987, 2003) . In contrast, if we neglect the business-cycle effect ($\tau_{BC} \approx 0$), we have $\tau = \tau_{Gap}$ where τ_{Gap} denotes the welfare costs of the business cycle linked to the consumption gap between average and steady-state consumption. We deduce that $(1-\tau) = (1-\tau_{BC})(1-\tau_{Gap}) \Rightarrow \tau \approx \tau_{BC} + \tau_{Gap}$. Numerical computations give τ and τ_{Gap} given $\mathbb{E}_0[C]$ and $\bar{C}, \mathbb{E}_0[U]$ and $\bar{U}$. The previous formula then gives τ_{BC}.

Table 2: Decomposition of welfare costs of business cycle

	Worker with financial frictions A	Worker without financial frictions B
Total welfare cost		
1. $\tau \times 100$	2.50	0.12
Decomposing the welfare cost		
2. $\tau_{Gap} \times 100$	2.23	0.06
3. $\tau_{BC} \times 100$	0.27	0.06

line 1 = line 2 + line 3

Welfare costs due to business-cycle fluctuations alone, in the spirit of Lucas' measure, τ_{BC}, are reported in Table 2, line 3, column A, and equal 0.27% of permanent consumption for workers. They are five times as large as in Lucas (1987, 2003) (0.05%). This first result comes from labor-market fluctuations (which are magnified by financial frictions). The latter are neglected by Lucas. In our model, τ_{BC} is a function of not only $Var(\widehat{c})$, as in Lucas, but also $Var(\widehat{u})$ and $Cov(\widehat{c}, \widehat{u})$, which, for a given "level effect", clearly magnify the costs of cycles. The most striking result is the measure of $\tau_{Gap} = 2.23\%$ in Table 2, line 2, column A. It accounts for the great increase in business-cycle costs: 90% of welfare costs come from this consumption gap. In fact, in Lucas (1987, 2003), $\tau_{Gap} = 0$. By assumption, there is no gap between average and steady-state consumption. Our model shows that this approximation is not acceptable because business-cycle volatility significantly affects the gap between average and steady-state employment and consumption. Thus, business-cycle costs are sizable: they are 50 times the amount estimated by Lucas.

Without financial frictions, workers' welfare cost falls drastically ($100 \times \tau = 0.12$, line 1, column B, Table 2).[47] The magnitude of business-cycle costs is reduced to 2.4 times Lucas's

[47]This estimate is computed by our model, with discounting heterogeneity and collateral constraints eliminated but the parameter values in Table 4 (panels (*b*) and (*c*)) retained. The rationale behind this approach is as follows. The model with financial frictions is considered the "true" model of the economy. By calibrating the model to match key financial and labor market targets, we uncover the "true" parameter values. The business-cycle costs without financial frictions are then computed with these parameter values, including the standard deviation of technological shock. If, in the model without financial frictions, we

evaluation. This estimation is slightly lower than the value reported in Hairault et al. (2010) or Jung & Kuester (2011). In our model, wages are set by using a Nash bargaining solution while Hairault et al. (2010) and Jung & Kuester (2011) assume exogenous wage rigidity or sluggishness. Finally, in the model without financial frictions, under a calibration that mimics the first-order allocation (i.e., $b = 0$, no unemployment allocation) and the Hosios condition $\epsilon = \psi$, welfare costs are negligible (0.02%). Welfare costs in the first-best economy are substantially lower than that which arises in presence of realistic financial and search and matching frictions.

If we apply our computations to the data, the contrast with Lucas's results is straightforward. The level effect associated with employment only is $100 \times \frac{\bar{N}-\mathbb{E}(N)}{\bar{N}} = 3.15$, i.e. a loss of 4.7 millions jobs in 2015 and a GDP per capita loss of approximately 1520 dollars a year. Without financial frictions, the level effect entails a loss of about 100.000 jobs, and each household loses 38 dollars per year.

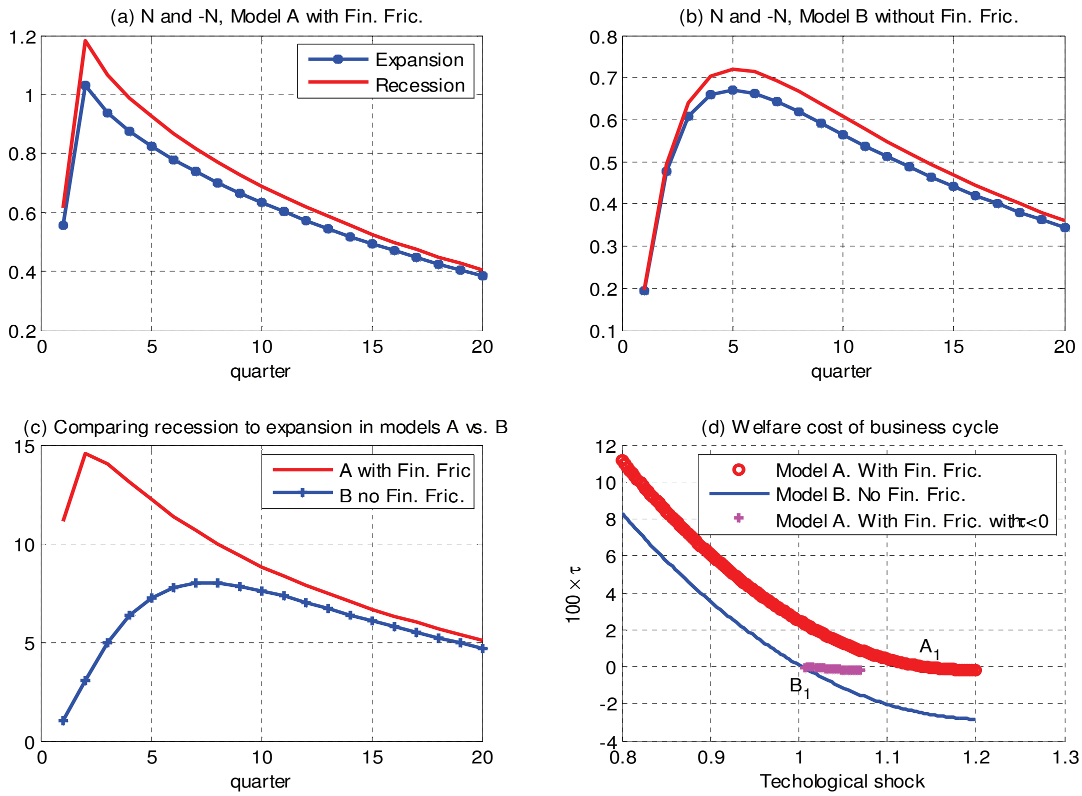
Asymmetric welfare costs of business cycles. As stressed by Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013), search and matching models feature asymmetric responses to business-cycle shocks; recessions (expansions) are characterized by severe and rapid rises (gradual decline) in unemployment. These asymmetric fluctuations are supported by empirical evidence.[48] The IRFs in model B (without financial frictions) are consistent with these features. Figure 6, panel (b), displays the employment response to the calibrated positive productivity shock when the economy is in boom – i.e., starting from a point where the economy is already hit by the same shock (dotted line)– and to a negative shock of the same magnitude when the economy is in recession (solid line). Employment falls more in a recession than it increases in a boom. Figure 6, panel (a), shows the IRFs from model A (with financial frictions) in a recession versus economic boom. The asymmetric response of employment is even larger than in model B. In order to measure this increased asymmetry due to the presence of financial frictions, we report, in panel (c), the gaps between IRFs in recession versus boom in both models. The gap between IRFs is twice as large in the economy with financial frictions (model A) as in the economy without financial frictions (model B); the maximum gap is around 15% in model A, but only 7.5%

adjust the standard deviation of technological shock to match output volatility, the welfare costs amount to 0.41% of permanent consumption, which remains far below the values reported in the model with financial frictions.

[48]See e.g. MacKay & Reis (2008) and Petrosky-Nadeau & Zhang (2013), Ferraro (2016) or Adjemian et al. (2016).

in model B.[49]. In addition, since labor market adjustments are connected to credit market conditions, the immediate response of credit market conditions directly affect employment dynamics. The maximum gap is reached after two quarters in model A and seven quarters in model B.

Figure 6: The Welfare cost of business cycles and asymmetry: Why do financial frictions matter?



(a): IRF of Employment ×100, N in response to a positive productivity shock starting from expansion (dotted line) and $-N$ in response to a negative productivity shock starting from recession (solid line) in model A with financial frictions. The size of the shock is calibrated as above. (b): IRF of Employment ×100, N in response to a positive productivity shock starting from expansion (dotted line) and $-N$ in response to a negative productivity shock starting from recession (solid line) in model B without financial frictions. The size of the shock is calibrated as above. (c) : $\frac{-IRF_{recession}-IRF_{expansion}}{IRF_{expansion}} \times 100$ in models A (solid line) and B (dotted line). (d) The steady-state value of technological shock is 1. Curves are quadratic fit over 30,000 simulations for model A (circle)and model B (solid line). + : segment of cost function for model A (only segment with $\tau < 0$, beyond point A_1) s hifted to point B_1.

In order to measure the implied business-cycle costs of these nonlinearities, we compute the time-varying welfare cost τ as in Petrosky-Nadeau & Zhang (2013) for model A (with financial frictions) and model B (without financial frictions).[50] Figure 6, panel (d), plots

[49]This last case summarizes the results reported in Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013).

[50]The welfare cost at each date is based on a comparison between the deterministic worker's welfare and the fluctuating worker's welfare summarized by his value function. This value function is time-varying. Its expected value is computed, for each current state of the economy, using the second-order Dynare approx-

the welfare cost $\tau \times 100$ against the technological shock. First, in model B, the welfare cost is countercyclical. In addition, welfare gains in expansion are much lower than welfare costs in recession, which is consistent with the asymmetric responses to business-cycle shocks displayed in matching models. Starting from point B_1, a 10% increase in productivity (from 1 to 1.1) results in a welfare gain of 2% while, for a fall in productivity of the same magnitude (from 1 to 0.9), the welfare loss is twice as large. Interestingly, welfare costs in model A are all located above the corresponding costs for model B, suggesting that the presence of financial frictions moves the economy to regions with higher unemployment levels and therefore higher welfare costs. Furthermore, since the welfare cost function shifts upwards, the business cycle starts being desirable ($\tau < 0$) only for large expansions (for a technological change of at least 1.15, beyond point A_1 in Figure 6). Even in the case of large expansions, the slope of the welfare cost function is nearly flat; welfare gains remain very small. In that sense, we argue that the presence of financial frictions makes the welfare costs more asymmetric than in the matching model. To illustrate this point, we report, in panel (d) of Figure 6, the welfare cost function of model A (just the segment beyond point A_1, with $\tau < 0$) at point B_1. It can then be easily seen that, when both models predict welfare gains (expansions), welfare gains remain small in model A, with financial frictions, while they increase quasi-linearly without financial frictions in model B.

5.4 Sensitivity analysis

In this section, we assess welfare costs of fluctuations for several extensions of the model. Table 3 reports the quantitative results. In column (2) of Table 3, we report the welfare

Table 3: Welfare costs : Sensitivity analysis

	benchmark	financial shocks	capital	capital and wage	financial shocks and capital
	(1)	(2)	(3)	(4)	(5)
$\tau \times 100$	2.50	1.79	2.67	3.7	3.4

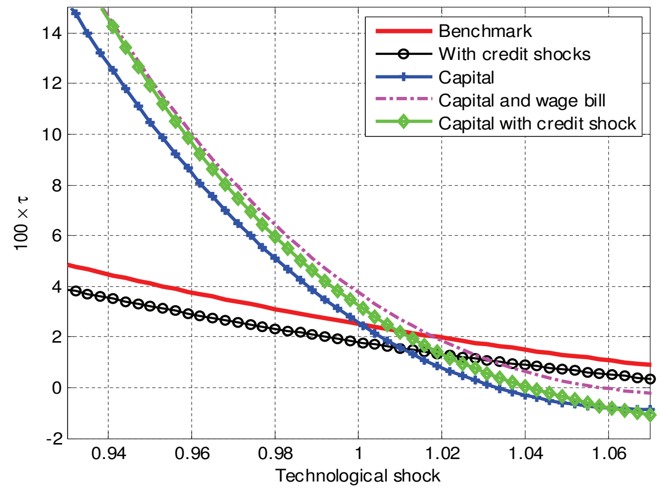
costs when the model is augmented with financial shocks. The latter are captured as shocks to parameter m in the collateral constraint (equation (9)), as in Jermann & Quadrini (2012) and Liu, Wang & Zha (2013). The calibration of the shock is based on Liu, Wang & Zha (2013).[51] Welfare costs in an economy with technological and financial shocks decrease to

imation of endogenous variables (see Adjemian et al. (2014)). We consider the decision rules around the mean, rather than the steady state, which is the default setting in Dynare

[51]See Appendix C.1 for more details.

1.79%. Because financial shocks introduce more uncertainty on the value that banks can recover from the collateral, the land price needs to incorporate a premium (average land price is greater than its steady-state value: $\mathbb{E}(q) > \overline{q}$). Thus, wealth increases, on average, raising consumption relative to the case with technological shocks only.[52] Financial shocks improve the match between debt and land-price volatilities (see Column (2) of Table 7 in the Appendix), but this shock induces excess volatility in vacancies. Figure 7 also shows that the asymmetries intrinsic to our model are not affected by the introduction of an additional shock. Indeed, the structure of our framework remains unaffected.

Figure 7: The Welfare cost of business cycles and asymmetry



The steady-state value of technological shock is 1. Curves are quadratic fit over 30,000 simulations for each model.

Column (3) of Table 3 incorporates capital into the model along the lines of Liu, Wang & Zha (2013).[53] Notice that welfare costs are greater than in the benchmark model. Indeed, capital amplifies our basic mechanism. As employment is lower, on average, than in steady state, the marginal productivity of capital is low in the stochastic economy, and thus the incentive to save is reduced. Notice also that when capital is included in the collateral constraint, models with only technological shocks are not able to replicate the volatilities of debt and land price (see column (3) of Table 7 in the Appendix) – making the assessment of the welfare cost of fluctuations less convincing.[54]

Welfare costs increase further when the collateral constraint also includes the wage bill; see column (4) of Table 3. As costs associated with hiring are greater in equilibrium, the

[52] See Appendix D.1 for a formal analysis of this mechanism using a simplified version of our model.
[53] See Appendix C.2 for more details.
[54] In Figure 7, the introduction of capital exacerbates the nonlinearities of the model (welfare gains and losses are larger than in the benchmark). As large gains are, on average, compensated by large losses, new asymmetries are not the main reason behind the change in average welfare costs when capital is introduced.

incentive to hold open vacancies is lower. In this modified framework, the financial wedge in the (JC) curve magnifies only the marginal product of labor (not net of the real wage, as in the benchmark model).

Finally, the introduction of both capital and financial shocks can be viewed as a solution to fit the financial indicators of the business cycle as well as those of investment (column (5) of Table 3). Indeed, this proceeds in the right direction, but at the cost of excess volatility on labor market fluctuations. This comes from the very low sensitivity of wages to the business cycle in this model; financial frictions are too strong and thus overestimate the wage moderation induced by the credit channel.[55] Notice also that the role of financial shocks in our model depends on whether we include capital. In economies without capital (columns (1) and (2) of Table 3), financial shocks reduce welfare costs. In contrast, in presence of capital (columns (3) and (5)), financial shocks increase welfare costs. This is because financial shocks make banks' return on the collateral more risky. This uncertainty generates a premium on the borrowing constraint (i.e., an over-accumulation). This in turn generates a new motive to increase debt (B increases in the volatile economy relative to the stabilized economy), entailing a parallel increase in workers' wealth. As a result, in our benchmark model, consumption is, on average, greater than at the steady state. This is not the case when we include capital accumulation. This latter setting is characterized by decreasing returns to scale. As greater leveraging in the volatile economy induce additional capital, over-accumulation is then costly in terms of consumption.[56]

6 Conclusion

This paper provides a quantitative assessment of welfare costs of fluctuations in a labor market search model with financial frictions *à la* Kiyotaki & Moore (1997). Because of labor market search frictions, fluctuations generate a higher average unemployment rate relative to its steady-state value, increasing the welfare cost of fluctuations. Financial frictions amplify this mechanism, together with the associated welfare costs. We show that business-cycle costs are sizable: they are 50 times the amount estimated by Lucas. Without financial constraints, the magnitude of business-cycle costs is reduced to 2.4 times Lucas's evaluation.

Moreover, our paper reveals significant asymmetries in the welfare response to business

[55] A solution for this shortcoming of this extension would be to consider a more complex modelling of the collateral constraint as in Liu, Wang & Zha (2013), where the collateral is not the simple sum of the value of capital and land, but a weighted sum of these two assets. Given that these weights are unknown, this is left for future research.

[56] See Appendix D.2 for an analysis of this mechanism in a Mickey-Mouse model.

cycles. These results suggest that structural policies aiming to remove financial frictions, per se, could have significant stabilizing macroeconomic effects. This is left for future research.

Our model is able to replicate business-cycle voltilities on both labor and financial markets. In particular, it reproduces the high degree of responsiveness of the job-finding rate throughout the business cycle. Indeed, financial frictions entail wage sluggishness that helps the model match the large changes in job-finding rates observed in the data; at the same time, it preserves the real wage volatility observed in the data.

References

Adjemian, S., Bastani, H., Karame, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. & Villemot, S. (2014), Dynare, Reference manual, Cepremap.

Adjemian, S., Karame, F. & Langot, F. (2016), On non-linearities in the unemployment, Mimeo, Cepremap.

Aiyagari, S. (1994), 'Uninsured idiosyncratic risk and aggrigate saving', *Quarterly Journal of Economics* pp. 659–684.

Andolfatto, D. (1996), 'Business cycles and labor-market search', *The American Economic Review* (1), 112–132.

Barron, J., Berger, M. & Black, D. (1997), 'Employer search, training, and vacancy duration', *Economic Inquiry* **35**, 167–192.

Barron, J. & Bishop, J. (1985), 'Extensive search, intensive search, and hiring costs: New evidence on employer hiring activity', *Economic Inquiry* **23**(3), 363–382.

Beaudry, P. & Pages, C. (2001), 'The cost of business cycles and the stabilization value of unemployment insurance', *European Economic Review* **45**, 1545–1572.

Becker, R., A. (1982), 'On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households.', *Quarterly Journal of Economics* **95**(2), 375–382.

Becker, R., A. & Foias, C. (1982), 'A characterization of ramsey equilibrium', *Journal of Economic Theory* **41**(1), 173–184.

Bernanke, B., Gertler, M. & Gilchrist, S. (1999), The financial accelerator in a quantitative business cycle framework, *in* J. B. Taylor & M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1, Elservier, pp. 1341–1393.

Blanchard, O. & Gali, J. (2010), ‘Labor markets and monetary policy: A new keynesian model with unemployment’, *American Economic Journal: Macroeconomics* **2**(2), 1–30.

Chéron, A. & Langot, F. (2004), ‘Labor market search and real business cycles: reconciling nash bargaining with the real wage dynamics’, *Review of Economic Dynamics* pp. 476–493.

Christiano, L., Eichenbaum, M. & Trabandt, M. (2016), ‘Unemployment and business cycle’, *Econometrica* (forthcoming).

Christiano, L., Motto, R. & Rostagno, M. (2010), Financial factors in economic fluctuations, ECB working paper 1192, ECB.

Dromel, N., Kolakez, E. & Lehman, E. (2010), ‘Credit constraints and the persistence of unemployment’, *Labour Economics* **17**.

Ferraro, D. (2016), The asymmetric cyclical behavior of the u.s. labor market, Mimeo, Arizona State University.

Garin, J. (2015), ‘Borrowing constraints,collateral fluctuations, and the labor market’, *Journal of Economic Dynamics and Control* **57**, 112–130.

Hagendorn, M. & Manovskii, I. (2008), ‘The cyclical behavior of equilibrium unemployment and vacancies revisited’, *American Economic Review* **98**(4), 1692–1706.

Hairault, J.-O., Langot, F. & Osotimehin, S. (2010), ‘Matching frictions, unemployment dynamics and the cost of business cycles’, *Review of Economic Dynamics* **13**(4), 759–779.

Hall, R. (2005), ‘Employment efficiency and sticky wages: Evidence from flows in the labor market’, *Review of Economics and Statistics* **87**(3), 397–407.

Hall, R. (2017), ‘High discounts and high unemployment’, *American Economic Review* .

Hall, R. & Milgrom, P. (2008), ‘The limited influence of unemployment on the wage bargain’, *American Economic Review* **98**(4), 1653–1674.

Iacoviello, M. (2005), ‘Housing prices, borrowing constraints and monetary policy in the business cycle’, *American Economic Review* **95**(3), 739–764.

Jermann, U. & Quadrini, V. (2012), ‘Macroeconomic effects of financial shocks’, *American Economic Review* **102**(1), 238–271.

Jung, P. & Kuester, K. (2011), 'The (un)importance of unemployment fluctuations for the welfare cost of business cycles', *Journal of Economic Dynamics and Control* **35**(10), 1744–1768.

Kiyotaki, N. & Moore, J. (1997), 'Credit cycle', *Journal of Political Economy* **105**(2), 211–248.

Krause, M. & Lubik, T. A. (2007), 'The (ir)relevance of real wage rigidity in the new keynesian model with search frictions', *Journal of Monetary Economics* **54**(4), 706–727.

Krebs, . (2003), 'Growth and welfare effects of business cycles in economies with idiosyncratic human capital risk.', *Review of Economic Dynamics* **6**, 846–868.

Krusell, P., Mukoyama, T. & Sahin, A. (2010), 'Labour-market matching with precautionary savings and aggregate fluctuations', *Review of Economic Studies* **77**(4), 1477–1507.

Krusell, P. & Smith, A. (1999), 'On the welfare effects of eliminating business cycles', *Review of Economic Dynamics* **2**, 245–272.

Langot, F. (1995), Unemployment and real business cycle : A matching model, *in* P. Henin, ed., 'Advances in Economic Business Cycles Theory', Springer Verlag.

Liu, Z., Miao, J. & Zha, T. (2013), Land prices and unemployment, NBER Working Paper 19382.

Liu, Z., Wang, P. & Zha, T. (2013), 'Land-price dynamics and macroeconomic fluctuations', *Econometrica* **81**(3), 1147–1184.

Lucas, R. (1987), *Models of Business Cycles*, Blackwell Publishing.

Lucas, R. (2003), 'Macroeconomic priorities', *American Economic Review* **93**(1), 1–14.

MacKay, A. & Reis, R. (2008), 'The brevity and violence of contractions and expansions', *Journal of Monetary Economics* **55**, 738–751.

Merz, M. (1995), 'Search in the labor market and the real business cycle', *Journal of Monetary Economics* **36**(2), 269–300.

Monacelli, T., Quadrini, V. & Trigari, A. (2011), Financial markets and unemployment, NBER Working Paper Series 17389, NBER.

Mortensen, D. & Pissarides, C. (1994), 'Job creation and job destruction in the theory of unemployment', *The Review of Economic Studies* **61**(3), 397–415.

Mulligan, C. (2012), Do welfare policies matter for labor market aggregates? quantifying safety network incentives since 2007, Working Paper 18088, NBER.

Petrongolo, B. & Pissarides, C. (2001), 'Looking into the black box : A survey of the matching function', *Journal of Economic Literature* **39**, 390–431.

Petrosky-Nadau, N. & Wasmer, E. (2013), 'The cyclical volatility of labor markets under frictional financial markets', *American Economic Journal: Macroeconomics* **5**(1), 193–221.

Petrosky-Nadeau, N. (2013), 'Credit, vacancies and unemployment fluctuations', *Review of Economic Dynamics* .

Petrosky-Nadeau, N. & Zhang, L. (2013), Unemployment crises, NBER Working Paper 19207, NBER.

Pissarides, C. (2009), 'The unemployment volatility puzzle: Is wage stickiness the answer?', *Econometrica* **77**(5), 1339–1369.

Pissarides, C. (2011), 'Equilibrium in the labor market with search frictions', *American Economic Review* **101**(4), 1092–1105.

Quadrini, V. (2011), 'Financial frictions in macroeconomic fluctuations', *Economic Quarterly* **97**(3), 209–254.

Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review* **95**(1), 25–49.

Shimer, R. (2012), 'Reassessing the ins and outs of unemployment', *Review of Economic Dynamics* **15**, 127–148.

Storesletten, K., Telmer, C. & Yaron, A. (2001), 'The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk', *European Economic Review* **45**, 1311–1339.

Wasmer, E. & Weil, P. (2004), 'The macroeconomics of labor and credit market imperfections', *American Economic Review* **94**(4), 944–963.

Zanetti, F. (2015), Financial shocks and labor market fluctuations, Working Paper 746, University of Oxford.

Zanetti, F. & Mumtaz, H. (2013), The effect of labor and financial frictions on aggregate fluctuations, Working Paper 690, University of Oxford.

Appendix For Online Publication

A Data

Aggregate data. The following quarterly time series come FRED database, the Federal Reserve Bank of Saint Louis' website (1976Q1-2013Q1). y is Real Gross Domestic Product from the FRED database (GDPC96) divided by the Civilian Non institutional Population from the FRED database (CNP16OV). c is Real Personal Consumption Expenditures from the FRED data-base (PCECC96) divided by the Civilian Non institutional Population from the FRED database (CNP16OV)

Labor market data. w is Compensation of Employees: Wages & Salary Accruals from the FRED database (WASCUR) divided by Civilian Employment (CE16OV). N is Civilian Employment (CE16OV) divided by Civilian Non institutional Population. U is FRED, Civilian Unemployment Rate (UNRATE), Percent, quarterly, Seasonally Adjusted. The latter time series are taken from the FRED database. As for the time series of the job finding rate, we use monthly CPS data from January 1976 to March 2013. We follow all the steps described in Shimer (2012). As in Shimer (2012), we correct for time aggregation and take quarterly averages of monthly observations. V are vacancies Total Nonfarm, Total US Job Openings (JTS00000000JOL), Seasonally Adjusted Monthly data from BLS. We take quarterly averages of this time series that is available only from December 2000 onwards.

Debt, loan Rate and land price. We follow Jermann & Quadrini (2012). Financial data come from the Flow of Funds Accounts of the Federal Reserve Board. The debt stock is constructed by using the cumulative sum of net new borrowing measured by the 'Net increase in credit markets instruments of non financial business'[57]. Since the constructed stock of debt is measured in nominal terms, it is deflated by the price index for business value added from NIPA. The initial (nominal) stock of debt is set to 94.12, which is the value reported in the balance sheet data from the Flow of Funds in 1952.I for the nonfarm non financial business. The cumulative sum starts in 1952, which, as in Jermann & Quadrini (2012), is not likely to affect our data starting on January 1976. R is the log of 1+ the Bank Prime Loan Rate (MPRIME) (used as a reference for short-term business loan) from

[57] Nonfinancial business; credit market instruments; liability; Net increase in credit markets instruments of non financial business, millions of dollars (nominal). FA144104005.Q, F.101 Line 28.

the FRED database. Finally, we use as a proxy for q the price index for residential land as computed by Liu, Wang & Zha (2013).

Cyclical components of the data: All data are quarterly (from 1976:Q1 through 2013:Q1), in logs, $HP(\lambda = 1600)$ filtered and multiplied by 100 in order to express them in percent deviation from steady state. Ψ is the job finding rate computed from Monthly CPS data from January 1976 to March 2013 using Shimer (2012)'s methodology. It measures the probability for an unemployed worker to find a job. As for financial data on debt and interest rate, we follow Jermann & Quadrini (2012). We finally check that our financial and labor market time series are consistent with the data available on line for Shimer (2012) and Jermann & Quadrini (2012).

Table 4: Calibration

(a) External information			
Notation	Label	value	Reference
β	discount factor (impatient)	0.99	Iacoviello (2005)
α	production function	0.99	Iacoviello (2005)
σ_W	risk aversion, worker	2	
σ_F	risk aversion, firm	1	Iacoviello (2005)
s	Job separation rate	0.1	Shimer (2005)
N	Employment	0.88	Hall (2005)
ψ	Elasticity of the matching function	0.5	Petrongolo & Pissarides (2001)
ω	cost of job posting	0.17	Barron et al. (1997) and Barron & Bishop (1985)
$\frac{b}{w}$	replacement ratio	0.6	OECD
A	average TFP	1	Normalization
ρ_A	Persistence	0.95	Hairault et al. (2010)
(b) Empirical target			
Notation	Label	value	Empirical target
μ	discount factor (patient)	$1/(1.04^{\frac{1}{4}})$	Annual real rate of 0.04
χ	scale parameter of matching function	0.634	Probability of filling a vacancy $\Phi = 0.95$
m	collateral constraint	0.61	corporate debt to GDP ratio $B/Y= 0.595$
σ_A	Standard deviation	0.0031	Observed σ_Y
(c) Derived parameter values			
Notation	Label	value	
Ψ	Job finding rate	0.423	
Γ	preference	0.29	

B Model

B.1 Household

Each household knows that the evolution of S follows (2), so that (5) can be written as:

$$N_t = (1-s)N_{t-1} + \Psi_t\left(1-(1-s)N_{t-1}\right) \tag{17}$$

The dynamic problem of a typical household can be written as follows

$$\mathcal{W}(\Omega_t^H) = \max_{C_t^n, C_t^u, B_t} \left\{ N_t U(C_t^n) + (1-N_t)U(C_t^u + \Gamma) + \mu \mathbb{E}_t \mathcal{W}(\Omega_{t+1}^H) \right\}$$

subject to (17) and (4), given the initial conditions on state variables (N_0, B_0) and $\Omega_t^H = \{N_{t-1}, \Psi_t, w_t, b_t, T_t, B_{t-1}\}$, the vector of variables taken as given by households. Let λ_t be the shadow price of the budget constraint. The first order conditions associated with consumption choices are

$$(C_t^n)^{-\sigma} = (C_t^u + \Gamma)^{-\sigma} = \lambda_t$$

Hence $U_t(C_t^n) = U_t(C_t^u + \Gamma)$. The first order condition associated to bond holdings reads:

$$-\lambda_t + \mu \mathbb{E}_t\left[R_t \lambda_{t+1}\right] = 0 \tag{18}$$

B.2 Entrepreneur

The firm's program is

$$\begin{aligned}
\mathcal{W}(\Omega_t^F) &= \max_{C_t^F, L_t, B_t, V_t, N_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t\left[\mathcal{W}(\Omega_{t+1}^F)\right]\right\} && (19)\\
s.t. & \left\{\begin{array}{rcll} -C_t^F - R_{t-1}B_{t-1} - q_t\left[L_t - L_{t-1}\right] - w_t N_t - \bar{\omega} V_t & & & \\ +Y_t\left(A_t, L_{t-1}, N_t\right) + B_t & = & 0 & (\lambda_t^F) \\ -B_t - \bar{\omega} V_t + m\mathbb{E}_t\left[q_{t+1} L_t\right] & = & 0 & (\lambda_t^F \varphi_t) \\ -N_t + (1-s)N_{t-1} + \Phi_t V_t & = & 0 & (\xi_t) \end{array}\right.
\end{aligned}$$

given the initial conditions N_0, B_0, where $\Omega_t^F = \{N_{t-1}, \Psi_t, w_t, b_t, \pi_t, T_t, B_{t-1}, L_{t-1}\}$ is the vector of variables taken as given by firms. Letting λ_t^F, $\lambda_t^F \varphi_t$, and ξ_t be the Lagrange multipliers associated to (7), (9) and (10) the first order conditions of problem (30) read:

$$U'\left(C_t^F\right) = \lambda_t^F \tag{20}$$

$$\lambda_t^F q_t = \beta \mathbb{E}_t\left[\lambda_{t+1}^F\left(q_{t+1}+\frac{\partial Y_{t+1}}{\partial L_t}\right)\right]+\lambda_t^F \varphi_t m \mathbb{E}_t\left[q_{t+1}\right] \tag{21}$$

$$(1-\varphi_t)\lambda_t^F = \beta \mathbb{E}_t \lambda_{t+1}^F R_t \tag{22}$$

$$\xi_t = \lambda_t^F \bar{\omega}\frac{(1+\varphi_t)}{\Phi_t} \tag{23}$$

$$\xi_t = \lambda_t^F\left[\left(\frac{\partial Y_t}{\partial N_t}\right)-w_t\right]+(1-s)\,\beta \mathbb{E}_t\left[\xi_{t+1}\right] \tag{24}$$

where (20) is the condition associated to consumption and (23) the one on vacancy posting[58]. Equation (C.2) is the one associated to land accumulation. It implies that, in equilibrium, the value of current marginal utility of consumption needs to equal the indirect value of utility deriving from land accumulation, i.e.: *i)* the value of future consumption utility deriving from reselling land in the next period, $\beta \mathbb{E}_t \lambda_{t+1}^F q_{t+1}$; *ii)* the future consumption utility arising from the product of land, $\beta \mathbb{E}_t \lambda_{t+1}^F \frac{\partial Y_{t+1}}{\partial L_t}$; *iii)* the additional utility arising from current consumption related to the effect of land in loosening the collateral constraint, $\varphi_t m \lambda_t^F \mathbb{E}_t\left[q_{t+1}\right]$.

Equation (22) is a modified Euler equation. When the collateral constraint is not binding, φ_t is equal to zero and we recover the standard Euler equation. When the debt limit is binding, $\varphi_t > 0$ and $\varphi_t = 1-\beta\frac{\mathbb{E}_t \lambda_{t+1}^F R_t}{\lambda_t^F}$ implying that firms' marginal utility of current consumption is greater than their discounted marginal utility of future consumption. Impatient firms choose thus to increase consumption up to the limit imposed by (9).

B.3 The wage curve

From the household's intertemporal program, one gets:

$$\begin{aligned}\mathcal{V}_t^H &= \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}} = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_t}\frac{\partial N_t}{\partial N_{t-1}} + \mu \mathbb{E}_t\left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\frac{\partial N_t}{\partial N_{t-1}} \\ &= \left[U_t(C_t^n) - U_t(C_t^u+\Gamma) + \lambda_t w_t - \lambda_t b_t - \lambda_t\left(C_t^n - C_t^u\right)\right]\frac{\partial N_t}{\partial N_{t-1}} + \mu \mathbb{E}_t\left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\frac{\partial N_t}{\partial N_{t-1}}\end{aligned}$$

With $U_t(C_t^n) = U_t(C_t^u+\Gamma)$, we have

$$\mathcal{V}_t^H = \left[\lambda_t w_t - \lambda_t(b_t+\Gamma)\right]\frac{\partial N_t}{\partial N_{t-1}} + \mu \mathbb{E}_t\left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\frac{\partial N_t}{\partial N_{t-1}}$$

[58]Note that, entrepreneurs are not risk neutral. By letting $\lambda_t^F = 1$ we recover the canonical search model.

Where, from (17) $\frac{\partial N_t}{\partial N_{t-1}} = (1-s)(1-\Psi_t)$, so that

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)(1-\Psi_t)\left[w_t - (b_t + \Gamma) + \mu \mathbb{E}_t\left(\frac{1}{\lambda_t}\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t}\right)\right] \tag{25}$$

From the firms' program $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}} = \xi_t (1-s)$ where $\xi_t = \lambda_t^F \bar{\omega}\frac{(1+\varphi_t)}{\Phi_t}$, thus:

$$\begin{aligned}\frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} &= (1-s)\frac{\bar{\omega}}{\Phi_{t+1}}\lambda_{t+1}^F (1+\varphi_{t+1}) \\ \frac{\mathcal{V}_t^F}{\lambda_t^F} &= (1-s)\frac{\bar{\omega}}{\Phi_t}(1+\varphi_t)\end{aligned}$$

Then, using (11) we obtain:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s)\left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta \mathbb{E}_t\left(\frac{1}{\lambda_t^F}\frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t}\right)\right]$$

Therefore, the surpluses are, respectively:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s)\left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta \mathbb{E}_t\left(\frac{\lambda_{t+1}^F}{\lambda_t^F}\frac{\mathcal{V}_{t+1}^F}{\lambda_{t+1}^F}\right)\right] \tag{26}$$

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)(1-\Psi_t)\left[w_t - (b_t + \Gamma) + \mu \mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_{t+1}}\frac{\mathcal{V}_{t+1}^H}{\lambda_t}\right)\right] \tag{27}$$

By maximizing the Nash product with respect to the wage, we obtain $\left(\frac{\mathcal{V}_t^H}{\lambda_t}\right) = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F}\right)\frac{(1-\epsilon)(1-\Psi_t)}{\epsilon}$. By substituting for (26) and (27), and rewriting it, we obtain the wage curve

B.4 The labor market in response to a productivity shock : analytical results

In what follows we investigate why and under which restrictions financial frictions entail the above-discussed greater responsiveness of labor-market tightness with respect to productivity, $\theta'(y)$. We then provide the set of restrictions that allow our model to solve the volatility puzzle of the DMP model. Finally, by using the quantitative predictions of our model, we show that the set of restrictions is satisfied at the general equilibrium. In order to provide straightforward intuitions, we are using a tractable version of the model incorporating the following assumptions.

Assumptions 1. *(1) Entrepreneurs' utility function is linear,* $\frac{\lambda_{t+1}^F}{\lambda_t^F} = 1$. *(2) The labor market is hit by one shock. Let* y_t *denote the marginal return of employment that follows* *the exogenous process* $\widehat{y}_t = \rho\widehat{y}_{t-1} + \varepsilon_t$ *where* $\widehat{y}_t$ *denotes* $\log(y_t/y)$. *(3) The tightness of the collateral constraint* φ_t *is given by the reduced form* $\widehat{\varphi}_t = -\frac{\beta}{\mu-\beta}\Lambda\varepsilon_t$.

Proposition 1. *Under Assumptions 1, and for reasonable restrictions on parameter values, financial frictions magnify labor-market-tightness fluctuations if* $\Lambda > \widetilde{\Lambda}$.

Proof. We characterize analytically the dynamics of the labor-market equilibrium in response to a productivity shock, under assumptions 1. More precisely, we restrict our analysis to an equilibrium where the value of search in the bargaining process is positive, and where the dynamics of θ are determined (saddle path). The values of parameters are to be considered "reasonable" as they imply usual properties of the labor market. Within these restrictions, our results are directly comparable to the standard DMP framework.

The system of equations of this problem is thus:

$$\begin{aligned}
\bar{\omega}\frac{(1+\varphi_t)}{\Phi_t} &= y_t - w_t + (1-s)\,\beta\mathbb{E}_t\left[\bar{\omega}\frac{(1+\varphi_{t+1})}{\Phi_{t+1}}\right] \\
w_t &= \epsilon(b+\Gamma) + (1-\epsilon)\left[y_t + \Sigma_t\right] \\
\Sigma_t &= (1-s)\mathbb{E}_t\left[\left(\frac{1+\varphi_{t+1}}{1-\varphi_t}\right)\beta\left\{-\frac{\bar{\omega}}{\Phi_{t+1}}\varphi_t + \bar{\omega}\theta_{t+1}\right\}\right] \\
\Phi_t &= \chi\theta_t^{\psi-1} \\
\varphi_t &= 1 - \frac{\beta}{\mu}(1+\Lambda\varepsilon_t)
\end{aligned}$$

while its log-linearized counterpart is:

$$\begin{aligned}
\frac{\varphi}{1+\varphi}\widehat{\varphi}_t - \widehat{\Phi}_t &= \frac{\Phi y}{\bar{\omega}(1+\varphi)}\widehat{y}_t - \frac{\Phi w}{\bar{\omega}(1+\varphi)}\widehat{w}_t + (1-s)\,\beta\mathbb{E}_t\left[\frac{\varphi}{1+\varphi}\widehat{\varphi}_{t+1} - \widehat{\Phi}_{t+1}\right] \\
\widehat{w}_t &= \frac{(1-\epsilon)y}{w}\hat{y}_t + \frac{(1-\epsilon)\Sigma}{w}\hat{\Sigma}_t \\
\widehat{\Sigma}_t &= \frac{\varphi}{1+\varphi}\mathbb{E}_t[\widehat{\varphi}_{t+1}] - \frac{\frac{\bar{\omega}}{\Phi}(\beta-\mu)}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\mathbb{E}_t[\widehat{\Phi}_{t+1}] \\
&\quad + \frac{\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\mathbb{E}_t[\widehat{\theta}_{t+1}] - \frac{\beta-\mu}{\beta}\frac{\frac{-\bar{\omega}}{\Phi}\mu+\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\widehat{\varphi}_t \\
\widehat{y}_t &= \rho\widehat{y}_{t-1} + \varepsilon_t \\
\widehat{\varphi}_t &= -\frac{\beta}{\mu-\beta}\Lambda\varepsilon_t \\
\widehat{\Phi}_t &= (\psi-1)\widehat{\theta}_t
\end{aligned}$$

where, using assumption, 1.3 $\mathbb{E}_t[\widehat{\varphi}_{t+1}] = 0$. Notice that for β sufficient close to μ, we obtain

- $-\frac{\frac{\bar{\omega}}{\Phi}(\beta-\mu)}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta} > 0 \Rightarrow$ a countercyclical component into the wage (because $\widehat{\Phi}_t < 0$ in booms).
- $-\frac{\beta-\mu}{\beta}\frac{\frac{-\bar{\omega}}{\Phi}\mu+\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta} < 0 \Rightarrow$ the pro-cyclical component into the wage (because $\widehat{\varphi}_t < 0$ in booms).

In order to gauge the relative weight of these two opposite mechanism, it is necessary to analyze them at the equilibrium. Hence, we solve the system so as to compute the effect of shocks on market tightness, θ:

$$\begin{aligned}\frac{\varphi}{1+\varphi}\widehat{\varphi}_t + (1-\psi)\widehat{\theta}_t &= \frac{\epsilon\Phi y}{\bar{\omega}(1+\varphi)}\widehat{y}_t + (1-s)\,\beta(1-\psi)\mathbb{E}_t\left[\widehat{\theta}_{t+1}\right] \\ &\quad - \frac{(1-\epsilon)\Phi\Sigma}{\bar{\omega}(1+\varphi)\left(\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta\right)}\left[\begin{array}{l}(1-\psi)\frac{\bar{\omega}}{\Phi}(\beta-\mu)\mathbb{E}_t[\widehat{\theta}_{t+1}] \\ +\mu\bar{\omega}\theta\mathbb{E}_t[\widehat{\theta}_{t+1}] - \frac{\beta-\mu}{\beta}\left(\frac{-\bar{\omega}}{\Phi}\mu+\mu\bar{\omega}\theta\right)\widehat{\varphi}_t\end{array}\right]\end{aligned}$$

This equation can be rewritten as:

$$A_1\widehat{\theta}_t = A_2\varepsilon_t + A_3\widehat{y}_t + A_4\mathbb{E}_t\left[\widehat{\theta}_{t+1}\right]$$

where the values of coefficients A_i are given in Table 5, whereas steady-state restrictions are reported in table 6.

Restriction 1: A positive value of search Σ in w. Without financial frictions, Σ is always larger than zero. With financial friction, one must restrict the analysis to β sufficiently close to μ, ie. $\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta > 0$. Hence, we have

$$\frac{\Sigma}{1+\varphi} = (1-s)\beta\left(-\frac{\bar{\omega}}{\Phi}\frac{\varphi}{1-\varphi} + \bar{\omega}\theta\frac{1}{1-\varphi}\right) = (1-s)\left(\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta\right) > 0$$

Table 5: Model coefficients: with versus without financial frictions

Coeff.	With financial frictions	No financial frictions
A_1	$1-\psi$	$1-\psi$
A_2	$-\frac{\beta}{\mu-\beta}\Lambda\left[\frac{(1-\epsilon)\Phi\Sigma}{\bar{\omega}(1+\varphi)}\frac{\beta-\mu}{\beta}\frac{\frac{-\bar{\omega}}{\Phi}\mu+\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta} - \frac{\varphi}{1+\varphi}\right]$	0
A_3	$\frac{\epsilon\Phi y}{\bar{\omega}(1+\varphi)}$	$\frac{\epsilon\Phi y}{\bar{\omega}}$
A_4	$(1-s)\,\beta(1-\psi) - \frac{(1-\epsilon)\Phi\Sigma}{\bar{\omega}(1+\varphi)}\left[\frac{\frac{(1-\psi)\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}{\frac{\bar{\omega}}{\Phi}(\beta-\mu)+\mu\bar{\omega}\theta}\right]$	$(1-s)\,\beta(1-\psi) - \frac{(1-\epsilon)\Phi\Sigma}{\bar{\omega}}$

Table 6: steady state values: with versus without financial frictions

Variable	With financial frictions	No financial frictions
$y - w$	$\frac{\bar{\omega}}{\Phi}(1+\varphi)\left[1-\beta(1-s)\right]$	$\frac{\bar{\omega}}{\Phi}[1-\beta(1-s)]$
w	$\epsilon(b+\Gamma)+(1-\epsilon)\left[y+\Sigma\right]$	
Σ	$(1-s)\left(\frac{1+\varphi}{1-\varphi}\right)\beta\left(-\frac{\bar{\omega}}{\Phi}\varphi+\bar{\omega}\theta\right)$	$(1-s)\beta\bar{\omega}\theta$
Φ	$\chi\theta^{\psi-1}$	$\chi\theta^{\psi-1}$
φ	$\frac{\mu-\beta}{\mu}$	0

Restriction 2: The saddle path. The dynamics of the model with financial friction are a saddle path, as the one without of financial friction iff

$$A_1\widehat{\theta}_t = A_2\varepsilon_t + A_3\widehat{y}_t + A_4\mathbb{E}_t\left[\widehat{\theta}_{t+1}\right] \Rightarrow \widehat{\theta}_t = a_1\varepsilon_t + \frac{a_2}{1-a_3\rho}\widehat{y}_t$$

where $a_1 = A_2/A_1$, $a_2 = A_3/A_1$, $a_3 = A_4/A_1$ and $a_3 < 1$. Given the steady-state values for Σ, term A_4 can be rewritten as

$$A_4 = \begin{cases} (1-s)\left[\epsilon\beta(1-\psi)+(1-\epsilon)\mu((1-\psi)-\Phi\theta)\right] & \text{with financial frictions} \\ (1-s)\beta\left[(1-\psi)-(1-\epsilon)\Phi\theta\right] & \text{without financial frictions} \end{cases}$$

Therefore, if $(1-\psi) > \Phi\theta = \Psi$, then $A_4 > 0$, with or without financial frictions. This condition is always satisfied for our calibration where $\psi = 0.5$ and $\Psi \approx 0.4$. Moreover, the solution is a saddle path iff $A_4/A_1 < 1 \Leftrightarrow a_3 < 1$, ie.

$$a_3 = \begin{cases} (1-s)\left[\epsilon\beta+(1-\epsilon)\mu\left(1-\frac{\Phi\theta}{1-\psi}\right)\right] < 1 & \text{with financial frictions} \\ (1-s)\beta\left[1-(1-\epsilon)\frac{\Phi\theta}{1-\psi}\right] < 1 & \text{without financial frictions} \end{cases}$$

These restrictions are always satisfied because $\frac{\Phi\theta}{1-\psi} < 1$ for $\psi = 0.5$ and $\Phi\theta = \Psi \approx 0.4$.

The overshooting behavior of θ with financial frictions. The instantaneous response of θ_t with respect to an innovation in the technology ε_t depends on $A_2 + A_3$. We compare $A_2 + A_3$ in the model with financial frictions to the corresponding term of the model without financial frictions. More precisely, we compare

$$(A_2+A_3)|_{\textit{with financial frictions}} = \Lambda\left[(1-s)(1-\epsilon)\mu\left(-1+\Phi\theta\right)+\frac{\beta}{2\mu-\beta}\right]+\frac{\epsilon\Phi y}{\bar{\omega}(1+\varphi)}$$

to $$(A_2+A_3)|_{\textit{without financial frictions}} = \frac{\epsilon\Phi y}{\bar{\omega}}$$

Financial frictions entail a multiplier of productivity shocks if:

$$(A_2 + A_3)|_{with\ financial\ frictions} > (A_2 + A_3)|_{without\ financial\ frictions}$$
$$\Rightarrow \Lambda > \frac{\mu - \beta}{2\mu - \beta}\epsilon\frac{\Phi}{\bar{\omega}}\frac{y}{(1-s)(1-\epsilon)\mu\left(-1+\Phi\theta\right)+\frac{\beta}{2\mu-\beta}} \quad (28)$$

where, using Tables 5 and 6, we deduce that $A_2 = \Lambda\left[(1-s)(1-\epsilon)\mu\left(-1+\Phi\theta\right)+\frac{\beta}{2\mu-\beta}\right] > 0$, iff β is sufficiently close to μ, so that $\frac{\beta}{2\mu-\beta}$ is sufficiently closed to one so as to compensate $(1-s)(1-\epsilon)\mu\left(-1+\Phi\theta\right) \in (-1;0)$.

The RHS of the inequality (28) gives the threshold value $\widetilde{\Lambda}$ for Λ : for any $\Lambda > \widetilde{\Lambda}$, a model with financial frictions amplifies the short run impact of a technological shock. The solution for Λ is given by the general equilibrium model. □

This result provides theoretical foundations for the large volatility of Ψ as it is observed in the data.

C Sensitivity analysis

C.1 Financial shocks

As in Liu, Wang & Zha (2013) and Jermann & Quadrini (2012), the financial shock is captured as a shock on m. This is interpreted as shocks on the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. The financial shock follows the stochastic process

$$log(m_t) = (1-\rho_m)log(\overline{m}) + \rho_m log(m_{t-1}) + \epsilon_t^m$$

with the calibrated values from Liu, Wang & Zha (2013)'s estimation results ($\rho_m = 0.9804$ and $\sigma^{\epsilon^m} = 0.0112$). The standard deviation of technological innovation σ_A is adjusted to match the observed standard deviation of output. This calibration is used in column (2) Table 7.

C.2 Model with capital

Model. In the model with capital, households' behavior do not change. The introduction of producing capital alters the entrepreneurs' problem. As in Liu, Wang & Zha (2013), the production function is now

$$Y_t = A_t \left[L_t^{\phi} K_{t-1}^{1-\phi} \right]^{1-\alpha} N_t^{\alpha}$$

with K the stock of capital. Capital accumulation is subject to adjustment costs such that

$$K_t = (1-\delta) K_{t-1} + I_t + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \quad (\lambda_t^K)$$

with I_t the investment flow and Ω the scale parameter on adjustment costs. The entrepreneur's budget constraint is now

$$C_t^F + R_{t-1} B_{t-1} + q_t^k \left[K_t - (1-\delta) K_{t-1} \right] + q_t \left[L_t - L_{t-1} \right] + w_t N_t + \bar{\omega} V_t \leq Y_t + B_t + \pi_t \quad (\lambda_t) \quad (29)$$

with λ_t the Lagrange multiplier on equation (29) and q_t^k the price of capital in consumption units. The collateral constraint now includes capital

$$B_t + \omega V_t \leq m \left(\mathbb{E}_t \left[q_{t+1}^k \right] K_t + \mathbb{E}_t \left[q_{t+1} \right] L_t \right)$$

The firm's program is

$$\mathcal{W}(\Omega_t^F) = \max_{C_t^F, L_t, B_t, V_t, N_t, K_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t \left[\mathcal{W}(\Omega_{t+1}^F) \right] \right\} \quad (30)$$

$$s.t. \begin{cases} -C_t^F - R_{t-1} B_{t-1} - q_t \left[L_t - L_{t-1} \right] - w_t N_t - \bar{\omega} V_t & \\ -q_t^k \left[K_t - (1-\delta) K_{t-1} \right] + Y_t \left(A_t, K_{t-1}, L_t, N_t \right) + B_t = 0 & (\lambda_t^F) \\ -B_t - \bar{\omega} V_t + m \mathbb{E}_t \left[q_{t+1}^k \right] K_t + m \mathbb{E}_t \left[q_{t+1} \right] L_t = 0 & (\lambda_t^F \varphi_t) \\ -N_t + (1-s) N_{t-1} + \Phi_t V_t = 0 & (\xi_t) \\ -K_t + (1-\delta) K_{t-1} + I_t + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t = 0 & (\lambda_t^K) \end{cases}$$

Let us define the shadow price of capital in consumption units

$$q_t^k = \frac{\lambda_t^K}{\lambda_t^F}$$

then the FOCs with respect to K_t is

$$q_t^k = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} \left((1-\alpha)(1-\phi) \frac{Y_{t+1}}{K_t} + q_{t+1}^k (1-\delta) \right) \right] + \varphi_t m \mathbb{E}_t \left[q_{t+1}^k \right]$$

and the choice of investment is such that

$$1 = q_t^k \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \frac{I_t}{I_{t-1}} \Omega \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} q_{t+1}^k \Omega \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]$$

Calibration. The capital adjustment cost parameter $\Omega = 0.1881$ and $\phi = 0.0695$ are set to the estimated values found in Liu, Wang & Zha (2013). $\delta = 0.025$ as in Liu, Wang & Zha (2013). $\alpha = 0.805$ is set so as to mimic the capital-output ratio (4.15 in the US data, as reported in Liu, Wang & Zha (2013)). $m = 0.123$ is chosen to match the benchmark value of $B/Y = 0.59$. Finally, the standard deviation of technological innovation σ_A is adjusted to match the observed standard deviation of output. This calibration is used in columns (3) and (5) of Table 7.

C.3 Model with capital and wage bill in the collateral constraint

Model. As previously, the behaviors of the households do not change. But as in Liu, Wang & Zha (2013), we introduce wage payment in the borrowing constraint. The wage bill needs to be financed by working capital such that the collateral constraint becomes

$$B_t + \omega V_t + w_t N_t \leq m \left(\mathbb{E}_t \left[q_{t+1}^k \right] K_t + \mathbb{E}_t \left[q_{t+1} \right] L_t \right)$$

The job creation curve becomes

$$\bar{\omega} \frac{(1 + \varphi_t)}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t (1 + \varphi_t) + (1 - s) \beta \frac{\bar{\omega}}{\lambda_t^F} \mathbb{E}_t \left[\lambda_{t+1}^F \frac{(1 + \varphi_{t+1})}{\Phi_{t+1}} \right]$$

The Nash bargaining is also altered such that the wage curve becomes:

$$w_t = \epsilon (b_t + \Gamma) + \frac{(1 - \epsilon)}{(1 + \varphi_t)} \frac{\partial Y_t}{\partial N_t} + (1 - s) \bar{\omega} (1 - \epsilon) \left[\begin{array}{c} \frac{\beta}{(1+\varphi_t)} \mathbb{E}_t \left(\frac{\lambda_{t+1}^F}{\lambda_t^F} \frac{(1+\varphi_{t+1})}{\Phi_{t+1}} \right) \\ +\mu \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{(\Psi_{t+1}-1)}{\Phi_{t+1}} \right) \end{array} \right]$$

Calibration. We consider the same calibration as in section C.2, except for $m = 0.28$ that is adjusted again to match the benchmark value of $B/Y = 0.59$. $\alpha = 0.81$ is set so as to mimic the capital-output ratio (4.15 in the US data, as reported in Liu, Wang & Zha (2013)). The standard deviation of technological innovation σ_A is adjusted to match the observed standard deviation of output. This calibration is used in column (4) of Table 7.

Table 7: Business-cycle volatility : Models versus data

	(0) Data		(1) Benchmark		(2) Financial shocks		(3) Capital		(4) Capital and wage		(5) Financial shocks and capital	
σ_A			0.0031		0.0024		0.0066		0.0071		0.0067	
	std(.)		std(.)		std(.)		std(.)		std(.)		std(.)	
Y	1.44	**	1.44	**	1.44	**	1.44	**	1.44	**	1.44	**
C	0.81	*	0.88	*	0.85	*	0.80	*	0.76	*	0.82	*
N	0.72	*	0.74	*	0.86	*	0.65	*	0.61	*	0.65	*
Y/N	0.54	*	0.28	*	0.21	*	0.52	*	0.57	*	0.52	*
w	0.62	*	0.49	*	0.50	*	0.35	*	0.31	*	0.36	*
U	7.90	*	5.45	*	6.30	*	4.81	*	4.51	*	4.80	*
Ψ	5.46	*	6.26	*	7.50	*	12.2	*	11.3	*	12.5	*
V	9.96	*	12.7	*	15.5	*	24.1	*	22.3	*	24.5	*
B	1.68	*	1.35	*	1.63	*	0.27	*	0.91	*	1.03	*
q	3.21	*	2.59	*	2.83	*	0.53	*	0.64	*	0.89	*
R	0.92	*	0.32	*	0.38	*	0.25	*	0.09	*	0.64	*
I	4.59	*		*		*	2.52	*	3.00	*	2.96	*
$corr(U,\Psi)$	-0.91		-0.86		-0.86		-0.54		-0.55		-0.54	
$corr(U,V)$	-0.97		-0.71		-0.72		-0.40		-0.40		-0.39	

** std (in percentage); * relative to GDP std

D Understanding the welfare effect of financial shocks using Mickey Mouse models

In this section, we show in Mickey Mouse models that financial shocks actually decrease welfare costs of fluctuations in a model with land only (section D.1) This provides a rationale for the quantitative results found in columns (1) and (2) in Table 3. Financial shocks actually increase business-cycle costs in a model with capital (section D.2), which explains the quantitative results found in columns (3) and (5) in Table 3

D.1 A simple model with land : Financial shocks decrease welfare costs

D.1.1 Household

The dynamic problem of a typical household can be written as follows

$$\mathcal{W}(\Omega_t^H) = \max_{C_t, B_t} \left\{ U(C_t) + \mu \mathbb{E}_t \mathcal{W}(\Omega_{t+1}^H) \right\} \quad \text{s.c.} \quad C_t + B_t \leq R_{t-1} B_{t-1} + w_t$$

given the initial conditions on state variables B_0 and $\Omega_t^H = \{w_t, B_{t-1}\}$, the vector of variables taken as given by households. Let λ_t be the shadow price of the budget constraint. The FOC are $(C_t)^{-\sigma} = \lambda_t$ and $-\lambda_t + \mu \mathbb{E}_t [R_t \lambda_{t+1}] = 0$. The labor supply is $N_t = 1$.

D.1.2 Entrepreneur

The firm's program is

$$\begin{aligned}
\mathcal{W}(\Omega_t^F) &= \max_{C_t^F, L_t, B_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t \left[\mathcal{W}(\Omega_{t+1}^F) \right] \right\} \\
s.t. & \quad \begin{cases} -C_t^F - R_{t-1} B_{t-1} - q_t [L_t - L_{t-1}] - w_t N_t + Y_t (A_t, L_{t-1}, N_t) + B_t = 0 & (\lambda_t^F) \\ -B_t + m_t \mathbb{E}_t [q_{t+1} L_t] = 0 & (\lambda_t^F \varphi_t) \end{cases}
\end{aligned}$$

given the initial conditions B_0, where $\Omega_t^F = \{w_t, B_{t-1}, L_{t-1}, m_t\}$ is the vector of variables taken as given by firms. Letting λ_t^F and $\lambda_t^F \varphi_t$ be the Lagrange multipliers associated to each constraint,the first order conditions of problem read:

$$\begin{aligned}
U'\left(C_t^F\right) &= \lambda_t^F \\
\lambda_t^F q_t &= \beta \mathbb{E}_t \left[\lambda_{t+1}^F \left(q_{t+1} + \frac{\partial Y_{t+1}}{\partial L_t} \right) \right] + \lambda_t^F \varphi_t m_t \mathbb{E}_t [q_{t+1}] \\
(1 - \varphi_t) \lambda_t^F &= \beta \mathbb{E}_t \lambda_{t+1}^F R_t \\
w_t &= \frac{\partial Y_t}{\partial N_t}
\end{aligned}$$

D.1.3 Equilibrium

Given that $N_t = 1$ and $L_t = 1\ \forall t$ and assuming that $Y_t = A_t L_t^{1-\alpha} N_t^{\alpha} = A_t$, we deduce

Dynamic system	Steady state
$1 = \mu \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\right] R_t$	$1 = \mu R$
$q_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F}\left(q_{t+1} + (1-\alpha)A_{t+1}\right)\right] + \varphi_t m_t \mathbb{E}_t\left[q_{t+1}\right]$	$q = \frac{\beta(1-\alpha)}{1-\beta-\varphi m} A$
$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F}\right] R_t + \varphi_t$	$1 = \frac{\beta}{\mu} + \varphi$
$w_t = \alpha A_t$	$w = \alpha A$
$C_t + B_t = R_{t-1} B_{t-1} + w_t$	$C = \frac{1-\mu}{\mu} B + \alpha A$
$A_t + B_t = C_t^F + R_{t-1} B_{t-1} + w_t$	$C^F = (1-\alpha)A - \frac{1-\mu}{\mu} B$
$B_t = m_t \mathbb{E}_t\left[q_{t+1}\right]$	$B = mq$

How does B change with m? This depends on $mq = \frac{m\beta(1-\alpha)}{1-\beta-\varphi m} A \equiv F(m, A)$. We have $F'_m = \frac{\beta(1-\alpha)(1-\beta)}{(1-\beta-\varphi m)^2} > 0$ whereas $F''_{mm} = \frac{2\varphi\beta(1-\alpha)(1-\beta)}{(1-\beta-\varphi m)^3} > 0$. Hence, this function is convex implying that $E[mq] > \overline{mq}$, which means that expected debt is larger than steady state debt: $E[B] > \overline{B}$. Given the worker's budget constraint ($C = \frac{1-\mu}{\mu} B + \alpha A$), we have $E[C] > \overline{C}$: expected consumption is larger than steady state consumption. The level effect on consumption actually implies that financial shocks in an economy with land are actually welfare-improving. Uncertainty generates a premium on the price of land as its return is risky for the bank. In this case, the value of worker's wealth increases on average, which explains the larger consumption in the stochastic economy, with respect to the stabilized economy.

How does B change with A? Given that F is linear in A, we deduce that $E[mq] = \overline{mq}$, hence $E[B] = \overline{B}$ and thus $E[C] = \overline{C}$. Technological shocks are neutral on the level effect of the welfare costs of the business cycle. This last result shows that welfare costs provided by our benchmark model are the result of the interaction between labor market and financial frictions, given that, without labor market frictions, the technological shocks are not costly in an economy with only financial constraints.

D.2 A simple model with capital : Financial shocks increase welfare costs

D.2.1 Individual behaviors

The household's program does not change. The firm's program becomes

$$\begin{array}{rcl} \mathcal{W}(\Omega_t^F) & = & \max_{C_t^F, B_t, K_t} \left\{ U\left(C_t^F\right) + \beta \mathbb{E}_t\left[\mathcal{W}(\Omega_{t+1}^F)\right]\right\} \\ s.t. & & \left\{ \begin{array}{rcll} -C_t^F - R_{t-1}B_{t-1} - w_t N_t - K_t + A_t K_{t-1}^{1-\alpha} N_t^{\alpha} + B_t & = & 0 & (\lambda_t^F) \\ -B_t + m_t K_t & = & 0 & (\lambda_t^F \varphi_t) \end{array}\right. \end{array}$$

The FOCs are

$$\begin{array}{rcl} U'\left(C_t^F\right) & = & \lambda_t^F \\ \lambda_t^F & = & \beta \mathbb{E}_t \left[\lambda_{t+1}^F \dfrac{\partial Y_{t+1}}{\partial K_t}\right] + \lambda_t^F \varphi_t m_t \\ (1-\varphi_t)\lambda_t^F & = & \beta \mathbb{E}_t \lambda_{t+1}^F R_t \\ w_t & = & \dfrac{\partial Y_t}{\partial N_t} \end{array}$$

D.2.2 Equilibrium

Given that $N_t = 1\ \forall t$, we deduce

$$\begin{array}{rcl|rcl} & \text{Dynamic system} & & & \text{Steady state} & \\ 1 & = & \mu \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right] R_t & 1 & = & \mu R \\ 1 & = & \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}^F}{\lambda_t^F}(1-\alpha)A_{t+1}K_t^{-\alpha}\right] + \varphi_t m_t & K & = & \left(\frac{\beta(1-\alpha)}{1-\varphi m} A\right)^{\frac{1}{\alpha}} \\ 1 & = & \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}^F}{\lambda_t^F}\right] R_t + \varphi_t & 1 & = & \frac{\beta}{\mu} + \varphi \\ w_t & = & \alpha A_t K_t^{1-\alpha} & w & = & \alpha A K^{1-\alpha} \\ C_t + B_t & = & R_{t-1}B_{t-1} + w_t & C & = & \frac{1-\mu}{\mu} B + \alpha A K^{1-\alpha} \\ A_t K_{t-1}^{1-\alpha} + B_t & = & C_t^F + R_{t-1}B_{t-1} + K_t + w_t & C^F & = & (1-\alpha)AK^{1-\alpha} - \frac{1-\mu}{\mu}B - K \\ B_t & = & m_t K_t & B & = & mK \end{array}$$

The steady state, conditional to $\{A, m\}$, gives $K = \mathcal{K}(m, A) = (\beta(1-\alpha)A)^{\frac{1}{\alpha}}(1-\varphi m)^{-\frac{1}{\alpha}}$ and $B = \mathcal{B}(m, A) = m\mathcal{K}(m, A)$. Assume a first restriction, which is satisfied in our calibration exercises, namely $C^F \approx 0$ ie. $\frac{1-\mu}{\mu}B \approx (1-\alpha)AK^{1-\alpha} - K$. We deduce that consumption C

is given by $C \approx A\mathcal{K}(m,A)^{1-\alpha} - \mathcal{K}(m,A)$,

$$\text{with} \quad \left\{ \begin{array}{rcl} \mathcal{K}'_m(m,A) & = & \frac{1}{\alpha}\frac{\varphi}{1-\varphi m}\mathcal{K}(m,A) > 0 \\ \mathcal{K}''_{mm}(m,A) & = & \frac{1+\alpha}{\alpha}\frac{\varphi}{1-\varphi m}\mathcal{K}'_m(m,A) > 0 \end{array} \right.$$

This leads to

$$C'' \quad = \quad \mathcal{K}'_m(m,A)\frac{\varphi}{1-\varphi m}\frac{1}{\alpha}\left[(1-\alpha)A\mathcal{K}(m,A)^{-\alpha} - (1+\alpha)\right]$$

Given that $\mathcal{K}'_m(m,A)\frac{\varphi}{1-\varphi m}\frac{1}{\alpha} > 0$, C'' has the same sign as the term between brackets, which consists of 2 terms. The first term $(1-\alpha)A\mathcal{K}(m,A)^{-\alpha} < 1$ because it represents an interest rate, and the second term $1+\alpha > 1$. This implies that $C'' < 0$, hence $E[C] < \overline{C}$ when the uncertainty comes from financial shocks. This shows that fluctuations in m reduce welfare.

Why do changes in m increase welfare costs when the collateral includes capital? In presence of financial shocks, the return on the collateral becomes risky for banks. This uncertainty generates a premium on the borrowing constraint (an over-accumulation). This generates a new motive to increase leveraging (B increases in the volatile economy relative to the stabilized economy) and thus, capital. As capital in our economy is characterized by decreasing returns to scale, this over-accumulation is then costly in terms of consumption.